Shahid
Beheshti
University

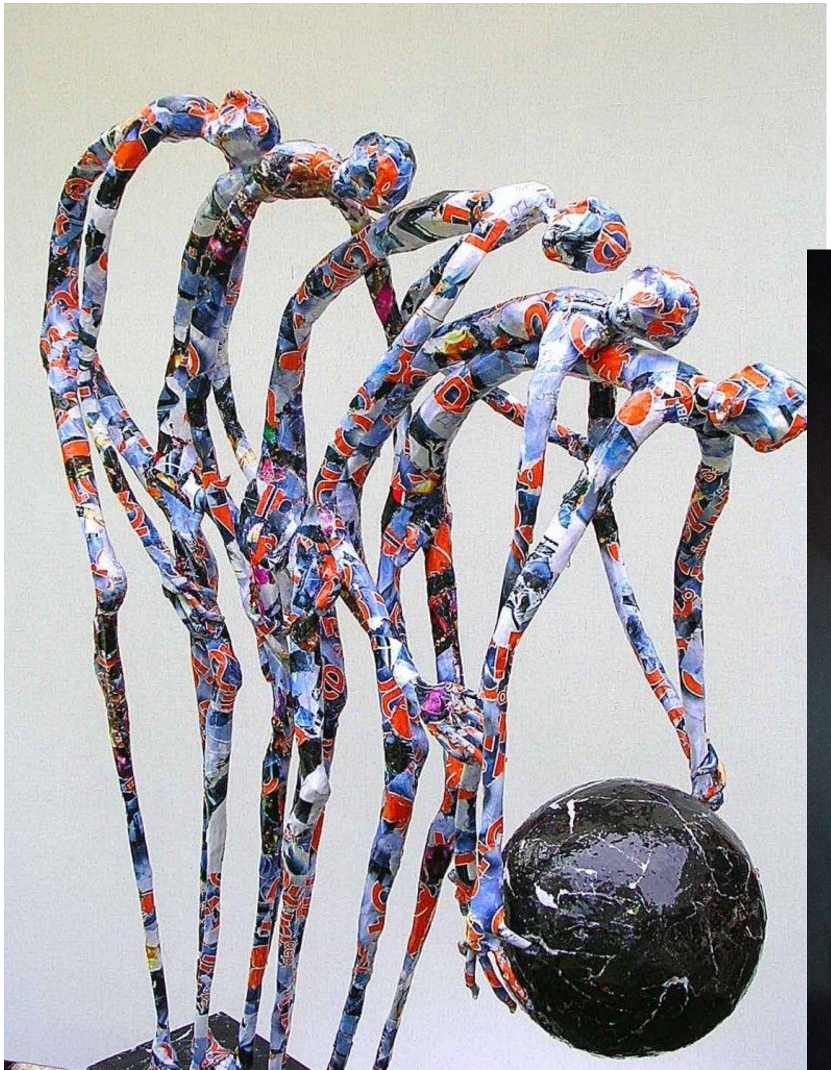
Frustration and aged networks dynamics

Reza Jafari
August 2018

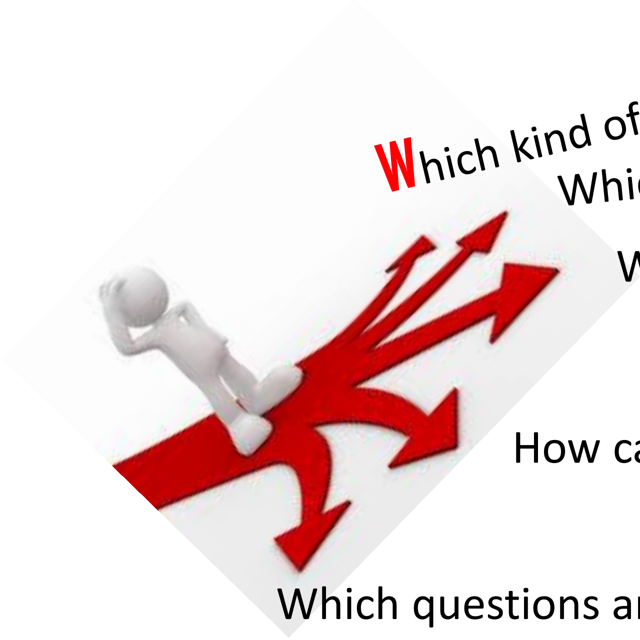
Tehran school on Theory and Applications of Complex Networks
25-29 August 2018

Jean-François Glabik modern sculpture artiste

Jean-François Glabik modern sculpture artiste


The skeleton of my talk



Which kind of networks are interested in my talk?

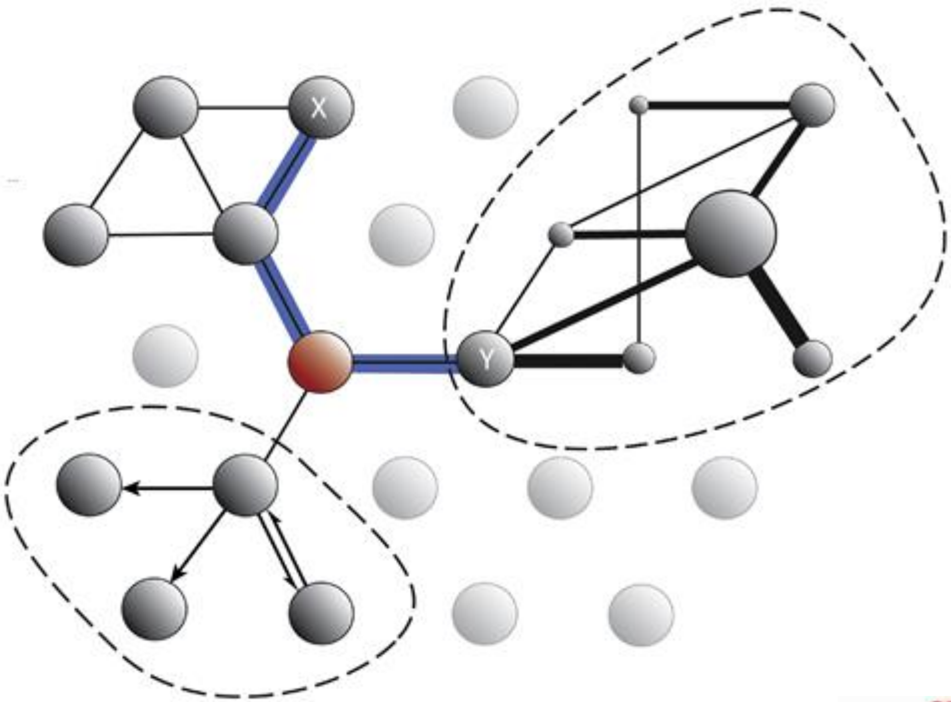
Which concepts do I would like to model?

Which questions I want to answer?

How can I answer to these questions?

Which questions are we going to answer in our future research?

Which kind of networks are interested in my talk?

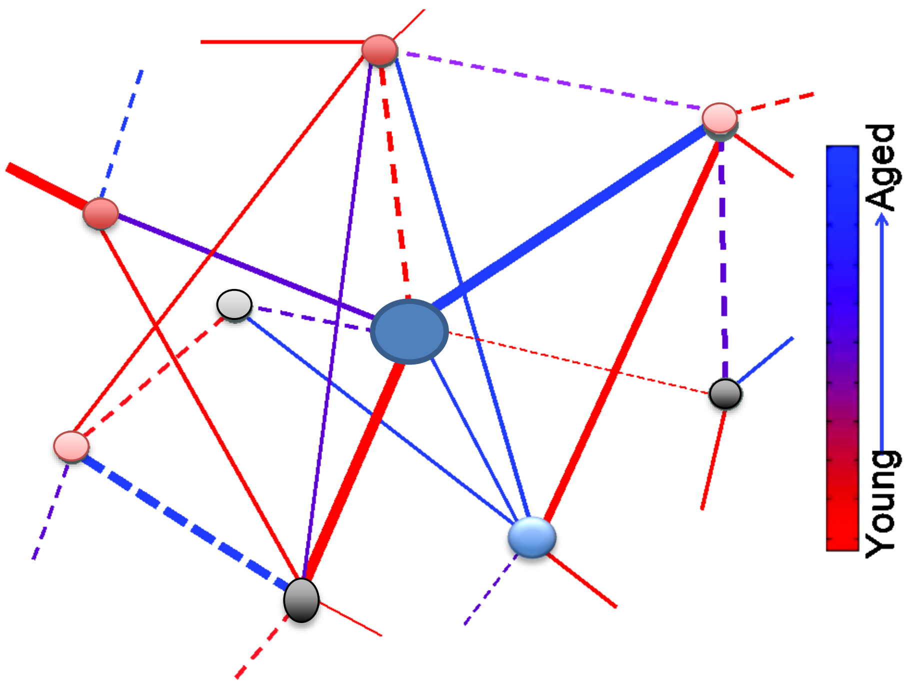


Weighed

Directed

Diameter, Mean length…

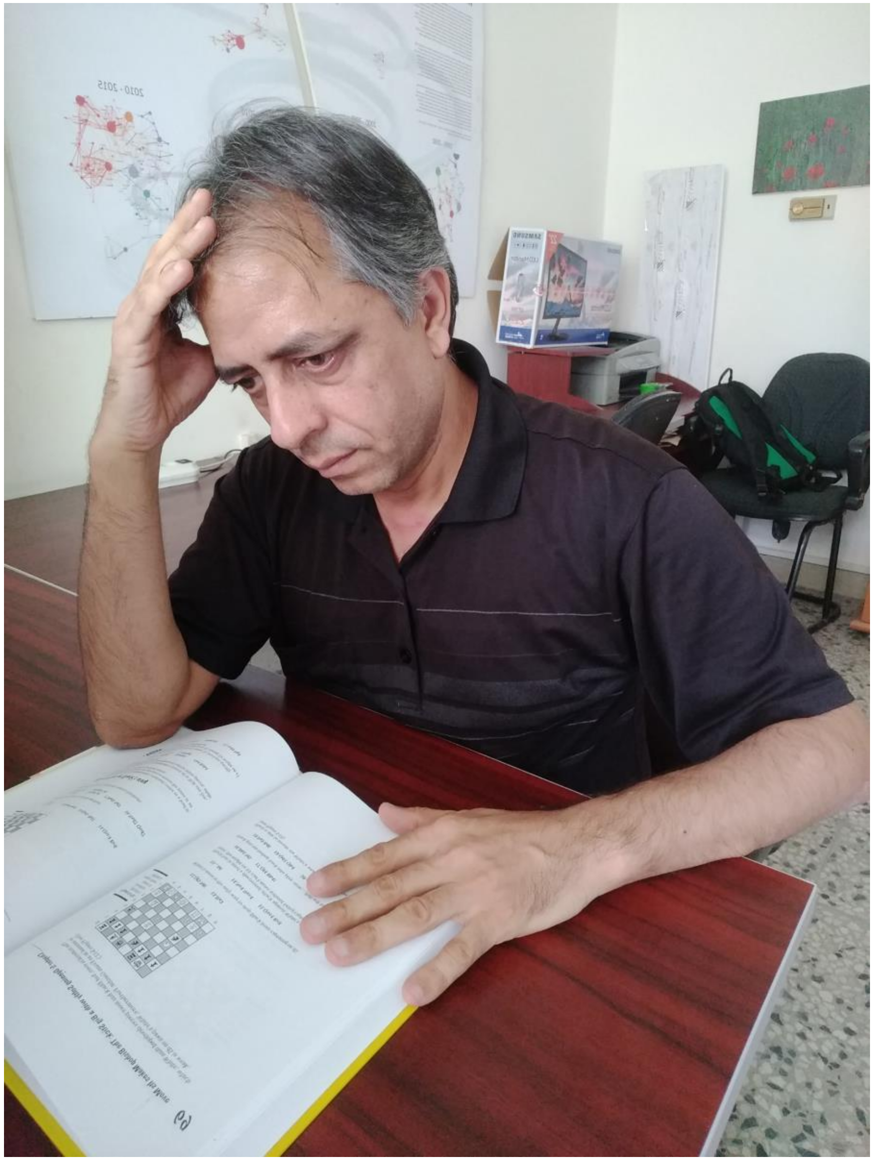
Signed & Aged links

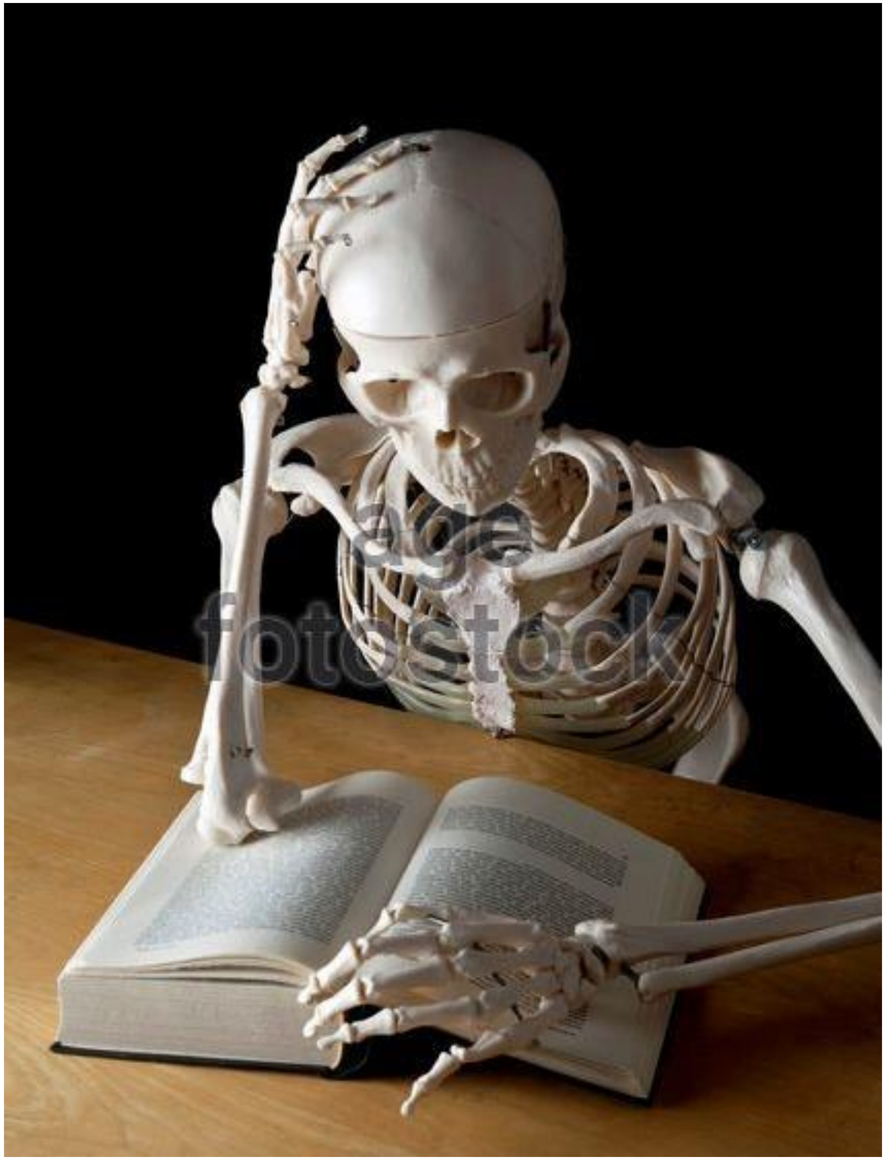


Fractional dynamics of network growth constrained by aging node interactions, H Safdari, MZ Kamali, A Shirazi, M Khalighi, G Jafari, M Ausloos, PloS one 11 (5), e0154983 (2016)

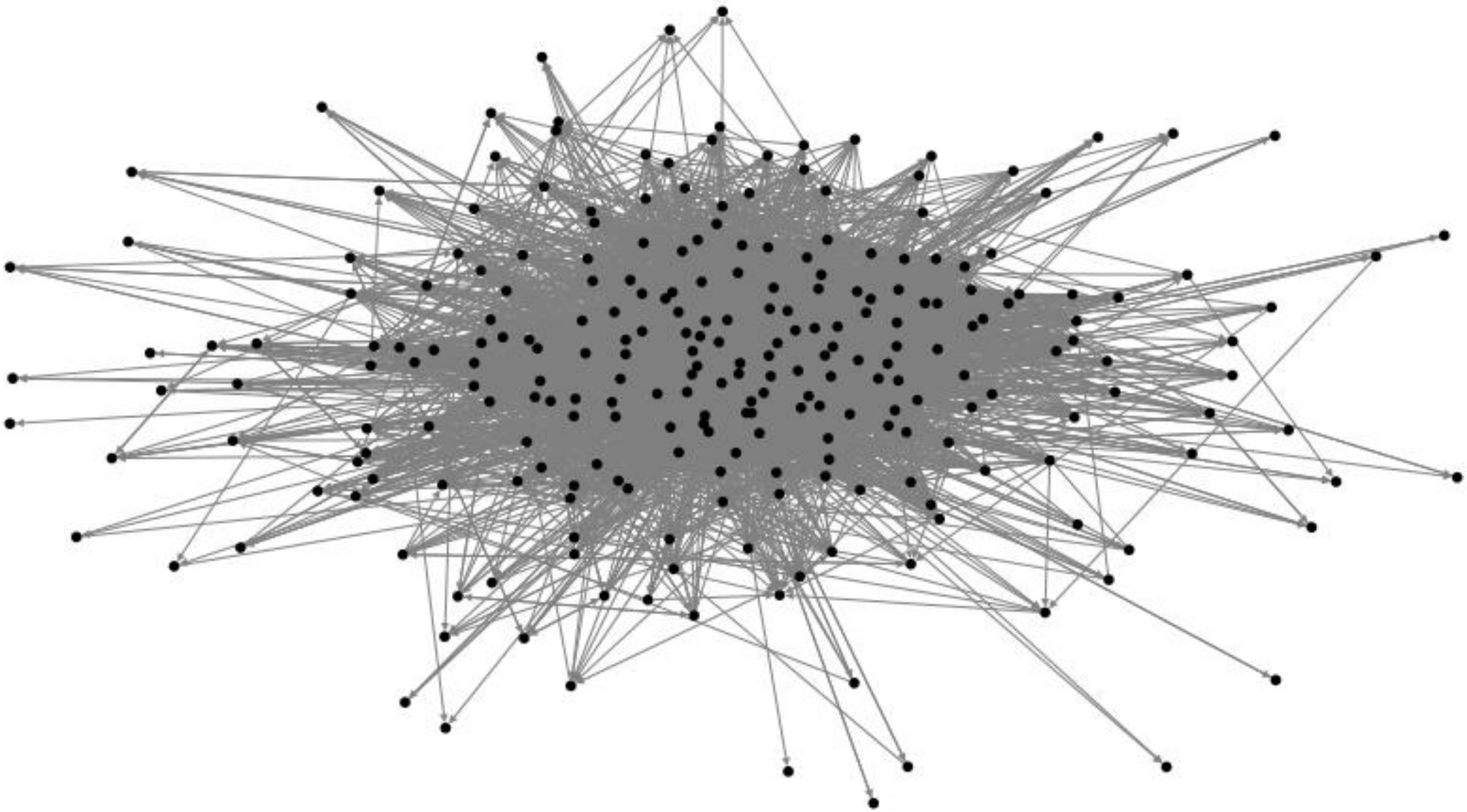
Glassy states of aging social networks, F Hassanibesheli, L Hedayatifar, H Safdari, M Ausloos, GR Jafari, Entropy 19 (6), 246 (2017)

My fake Skeleton

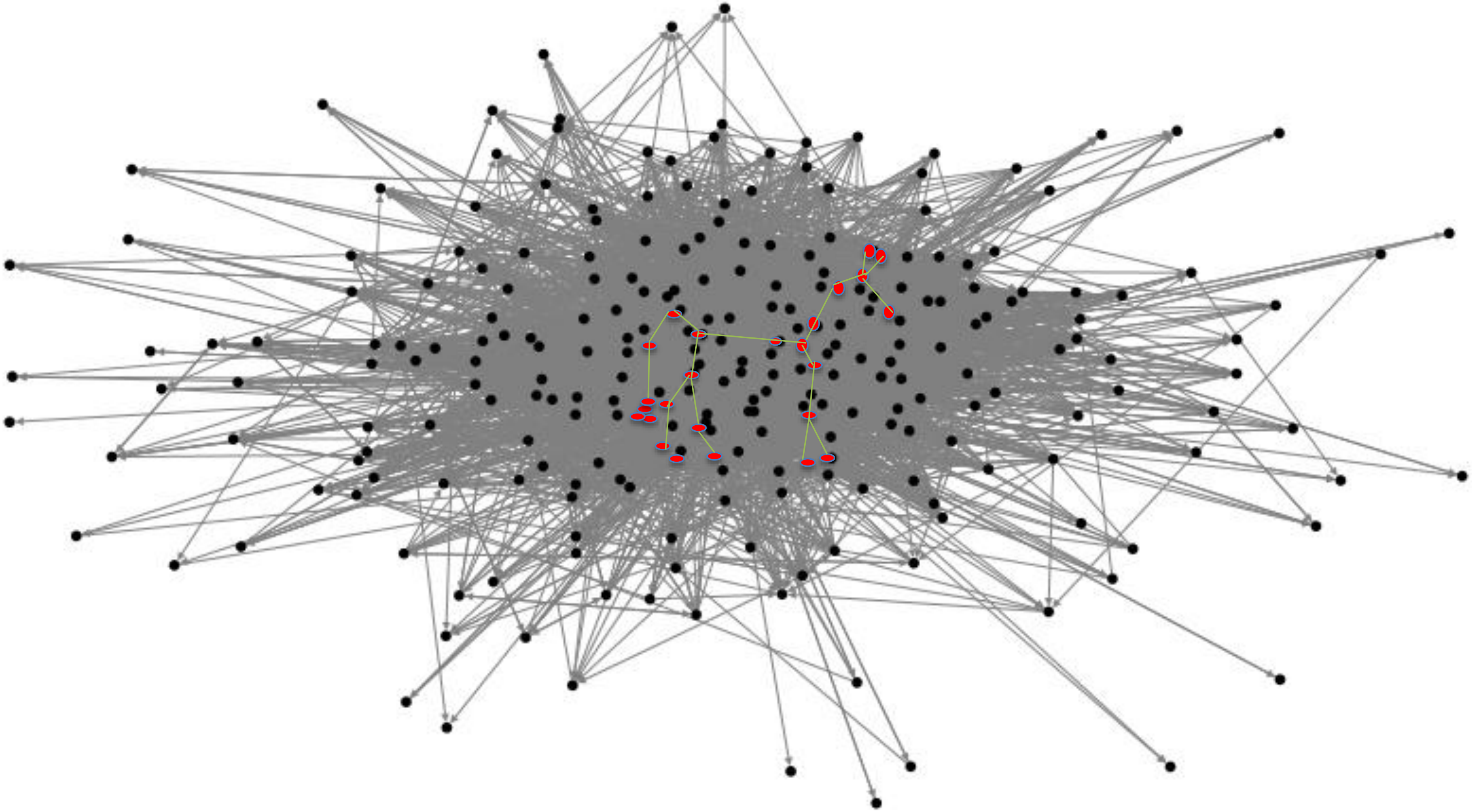




X- ray method on complex networks



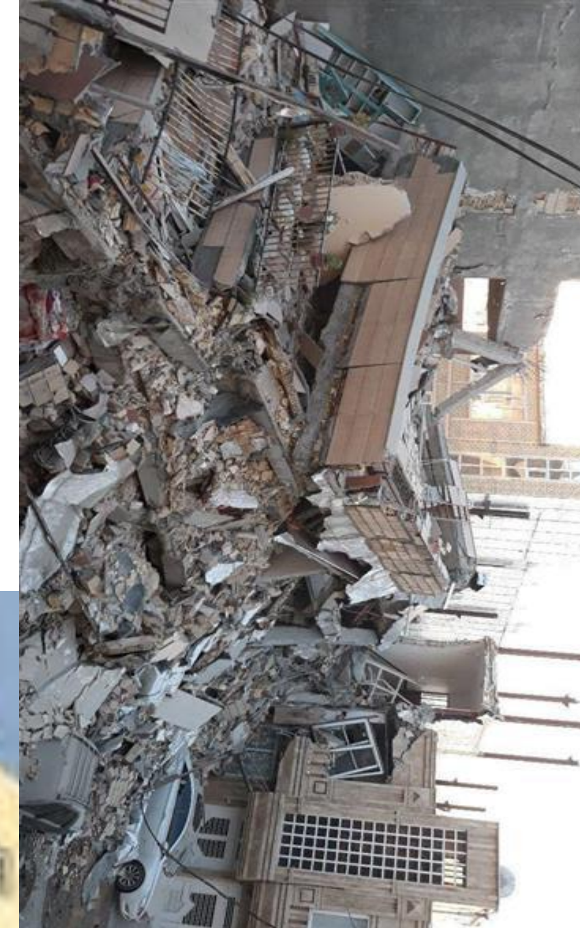
A modern horse network



One concept of real Social Networks

The relationships between two persons is not only their problem. Because we tie together. One reason to emerge a collective behavior.


photo : Mahmoud Hosseini

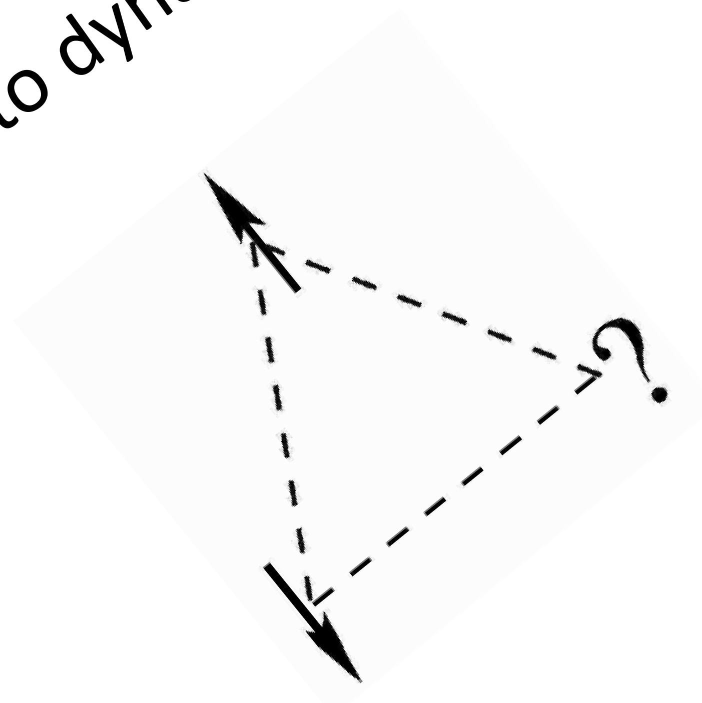


The second concept

Our history of experiences are effected on our future.

By emerging a new structure,

frustration is a secret key to dynamic and fluctuation.



Frustration
Cognitive Balance Theories

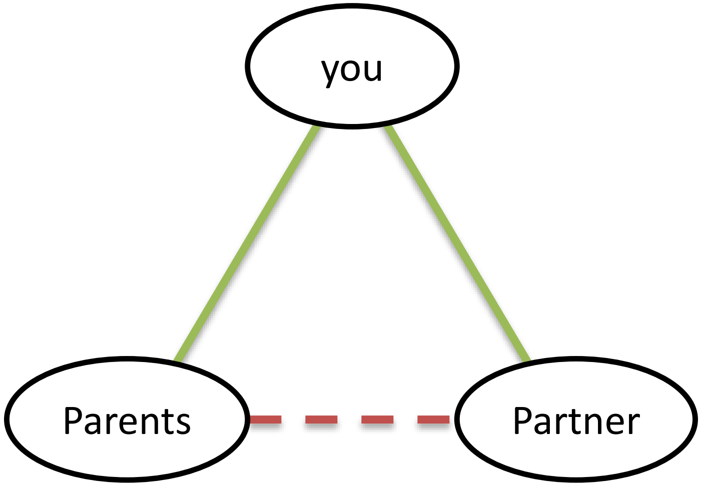
(Fritz Heider 1946)

We adjust our relationship based on reducing the psychological stresses

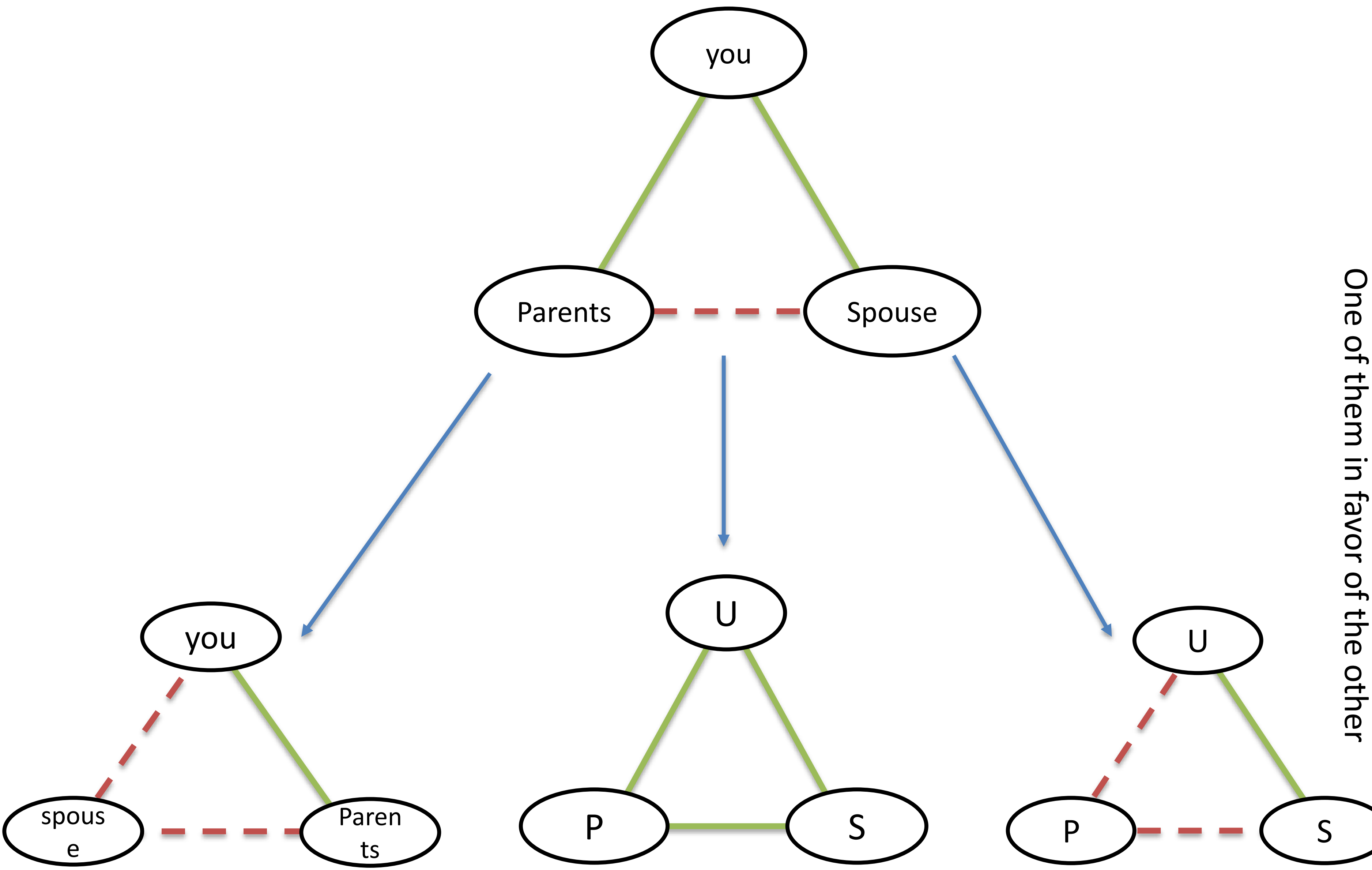
If a person's beliefs are unbalanced, psychological stresses will generate internal pressures to change either some of the sentiments (liking, disliking) or some relationships (proximity, membership) into a more congruent pattern.

The hard situation

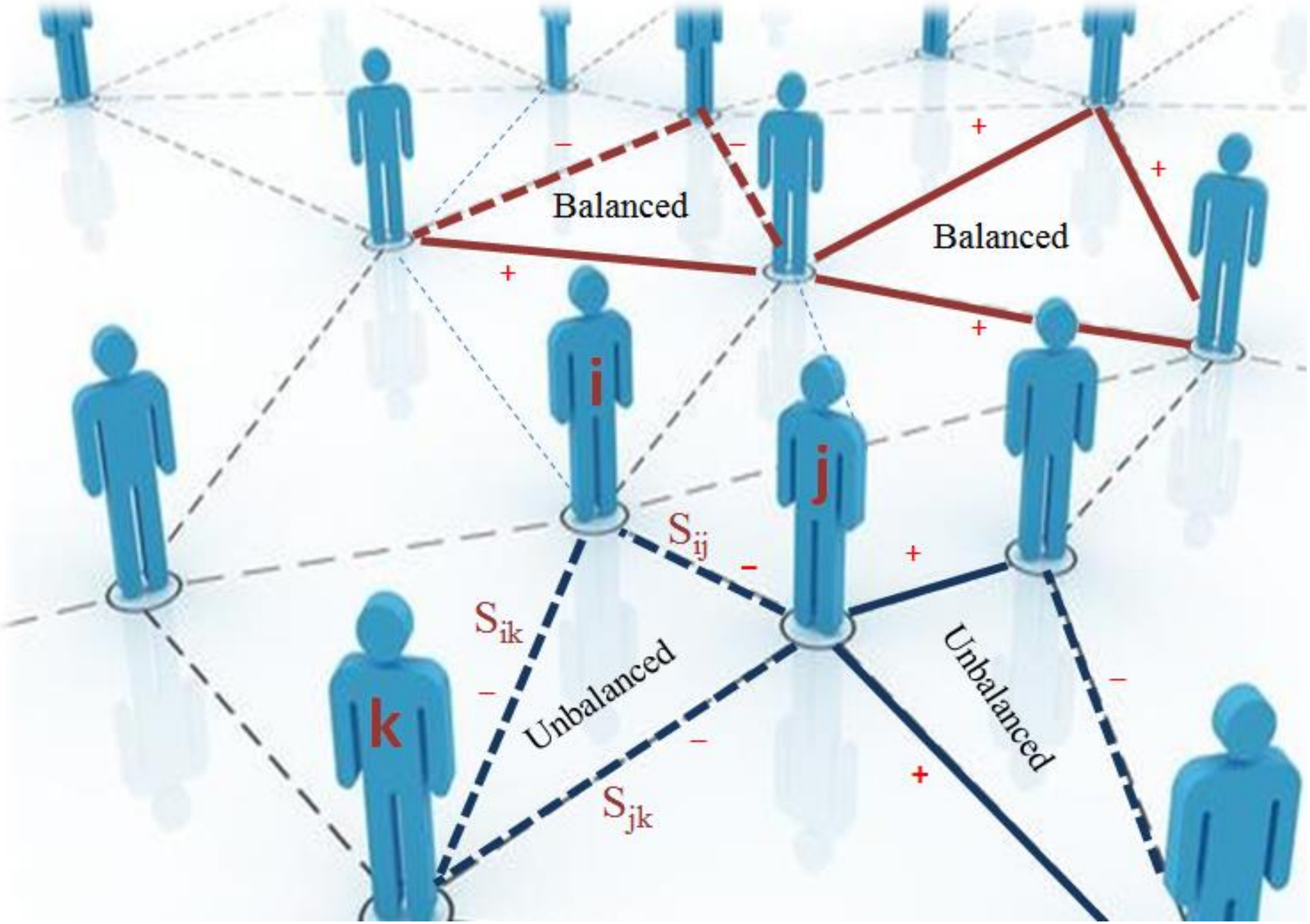
Cartwright and Harary (1950)



Balanced situations



Our relationships are effected from each other

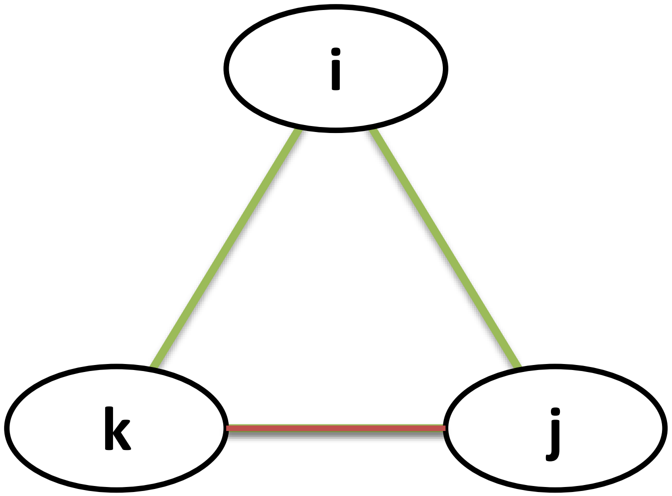


Mathematical model

$$U = -\frac{1}{\binom{n}{3}} \sum s_{ij} s_{jk} s_{ik}$$

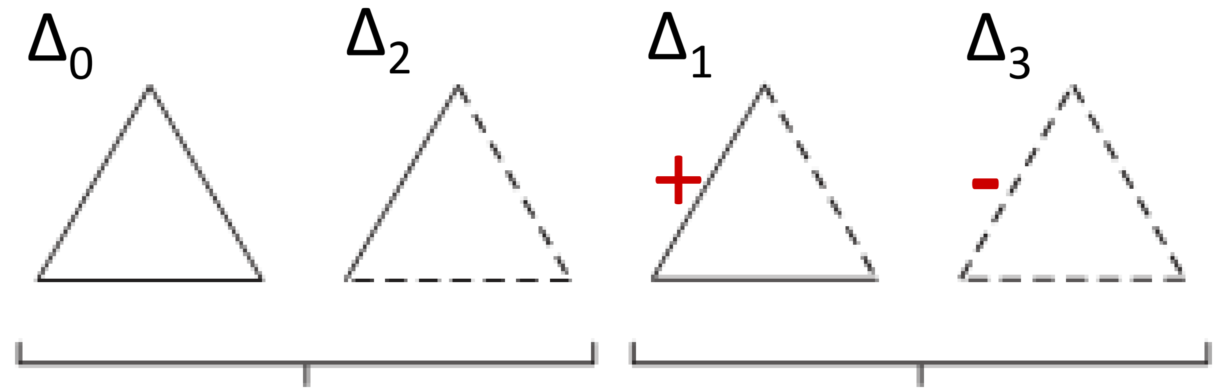
(Sum over all triangles)

$$s_{ij} = \begin{cases} +1 & \text{(friendship)} \\ -1 & \text{(enemy)} \end{cases}$$



Structural balance

$$H = -\frac{1}{\binom{n}{3}} \sum s_{ij} s_{jk} s_{ik}$$



balanced triangles	*unbalanced triangles*
$S_{ij}\ S_{jk}\ S_{ki}=1$	$S_{ij}\ S_{jk}\ S_{ki}=-1$
Even number of dashed edges	Odd number of dashed edges

$$\frac{dx_{ki}}{dt} = \sum_{l=1} X_{kl}(t') X_{li}(t')$$

Δ_2: An enemy of my friend is my enemy

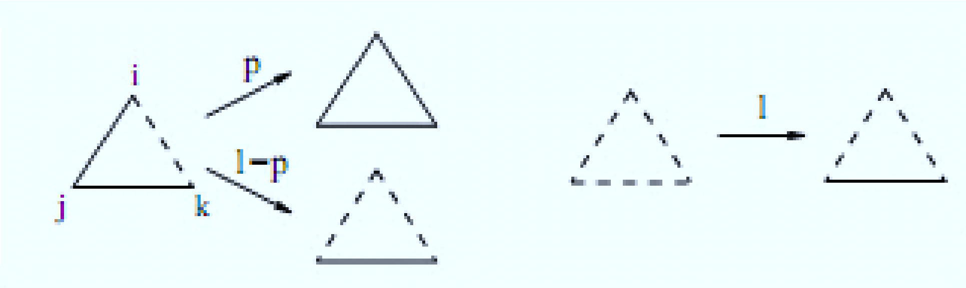
Δ_3: An enemy of my enemy is my enemy

T. Antal, P. L. Krapivsky, and S. Redner, Phy. Rev. E 72. 036121 (2005)

Local triad dynamic (LTD)

A random triad select
- If this triad is balanced, no evolution occurs.
- If this triad is imbalanced : S ⟶ -S

$$H = -\frac{1}{\binom{n}{3}} \sum s_{ij} s_{jk} s_{ik}$$



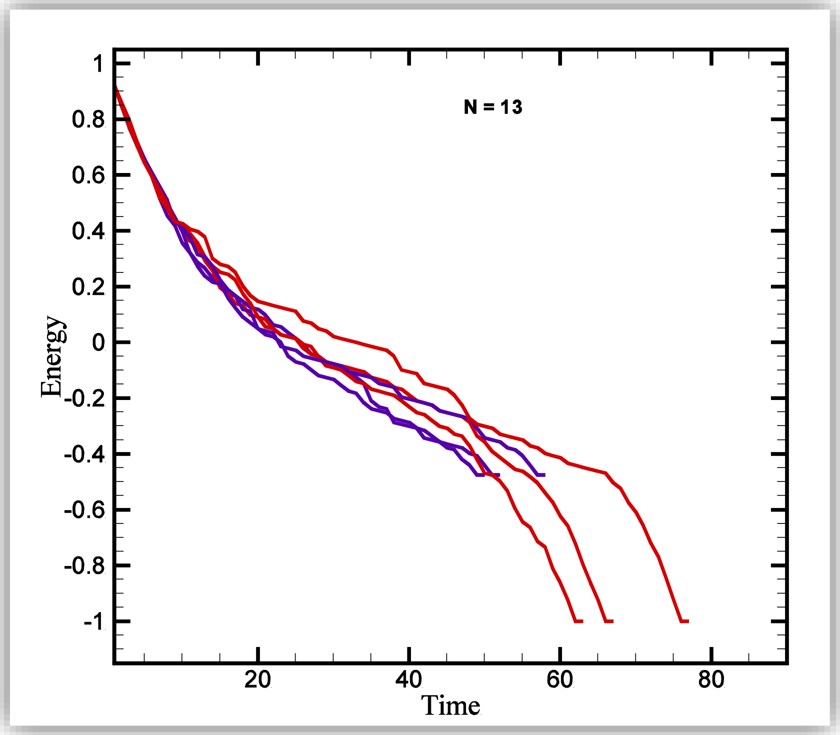
Constrained triad dynamic (CTD)

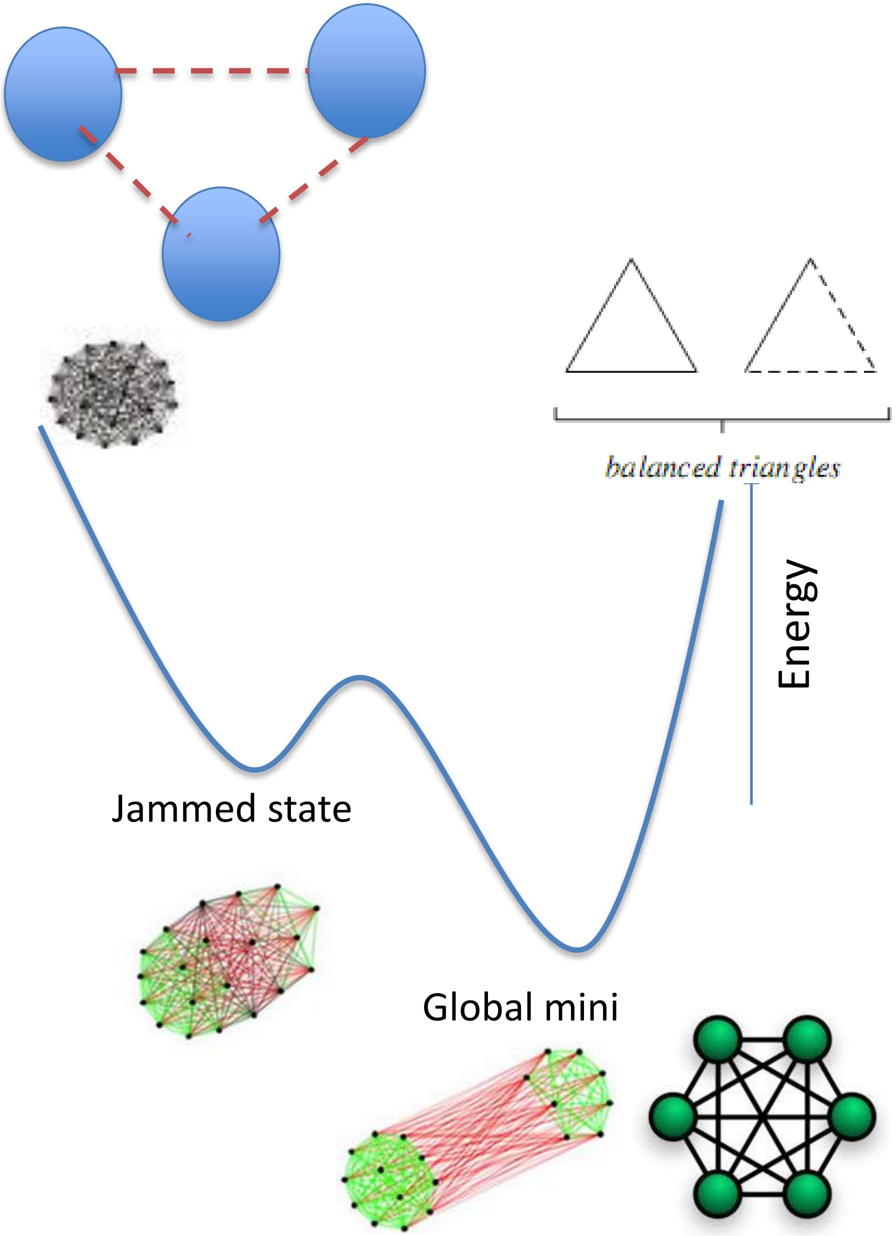
1. A random link select
2. S ⟶ -S , if the total number of imbalance triads decrease.
3. S ⟶ -S with probability 1/2 , if the total number of imbalance triads is conserved.

T. Antal, P. L. Krapivsky, and S. Redner, PRE 72, 036121 (2005)

Jammed states

$$H = -\frac{1}{\binom{n}{3}} \sum s_{ij} s_{jk} s_{ik}$$





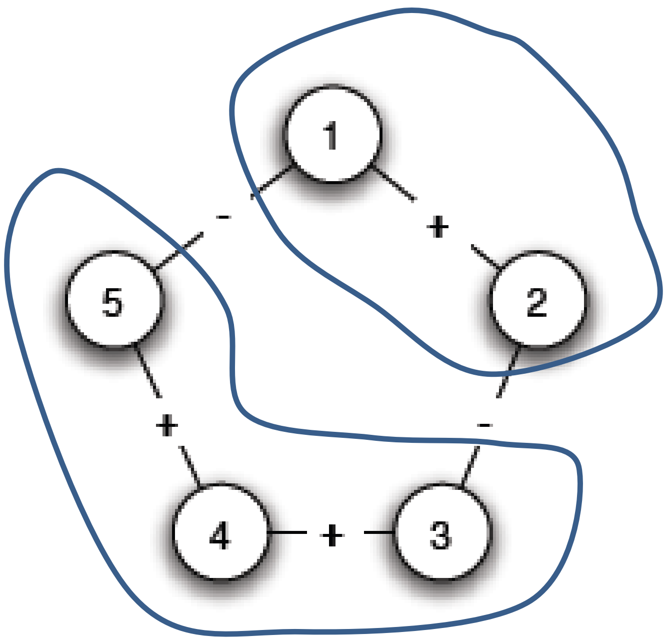
Jammed states are possible if and only if the network size is **N = 9 or N ≥ 11**

S. A. Marvel, S. H. Strogatz, and J. M. Kleinberg, Phys. Rev. Lett. 103, 198701 (2009)

Balance Definition for General Graphs

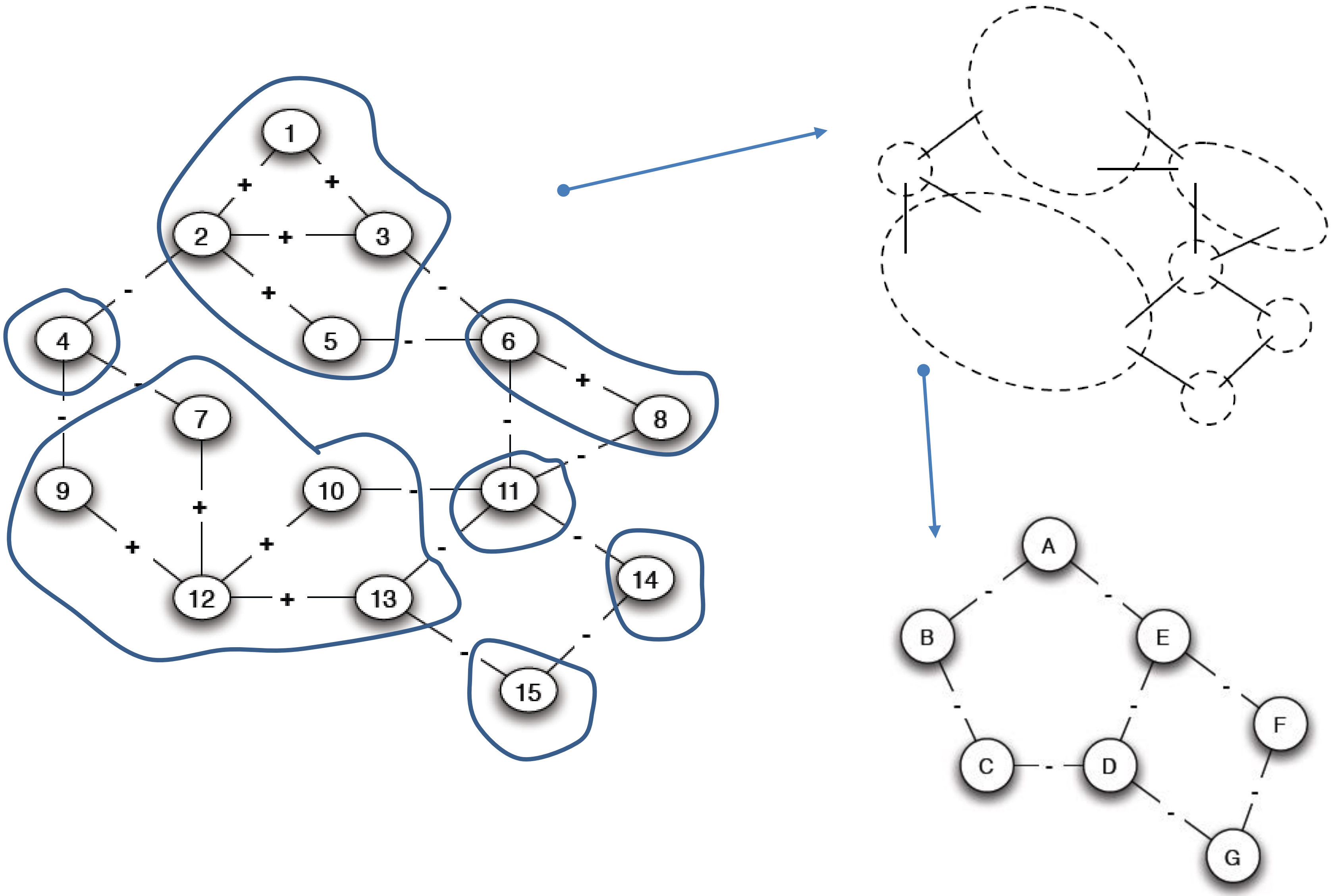
1. Based on triangles (local view)
2. Division of the network (global view)

A (non-complete) graph is balanced if it possible to divide the nodes into two sets X and Y, such that any edge with both ends inside X or both ends inside Y is positive and any edge with one end in X and one end in Y is negative



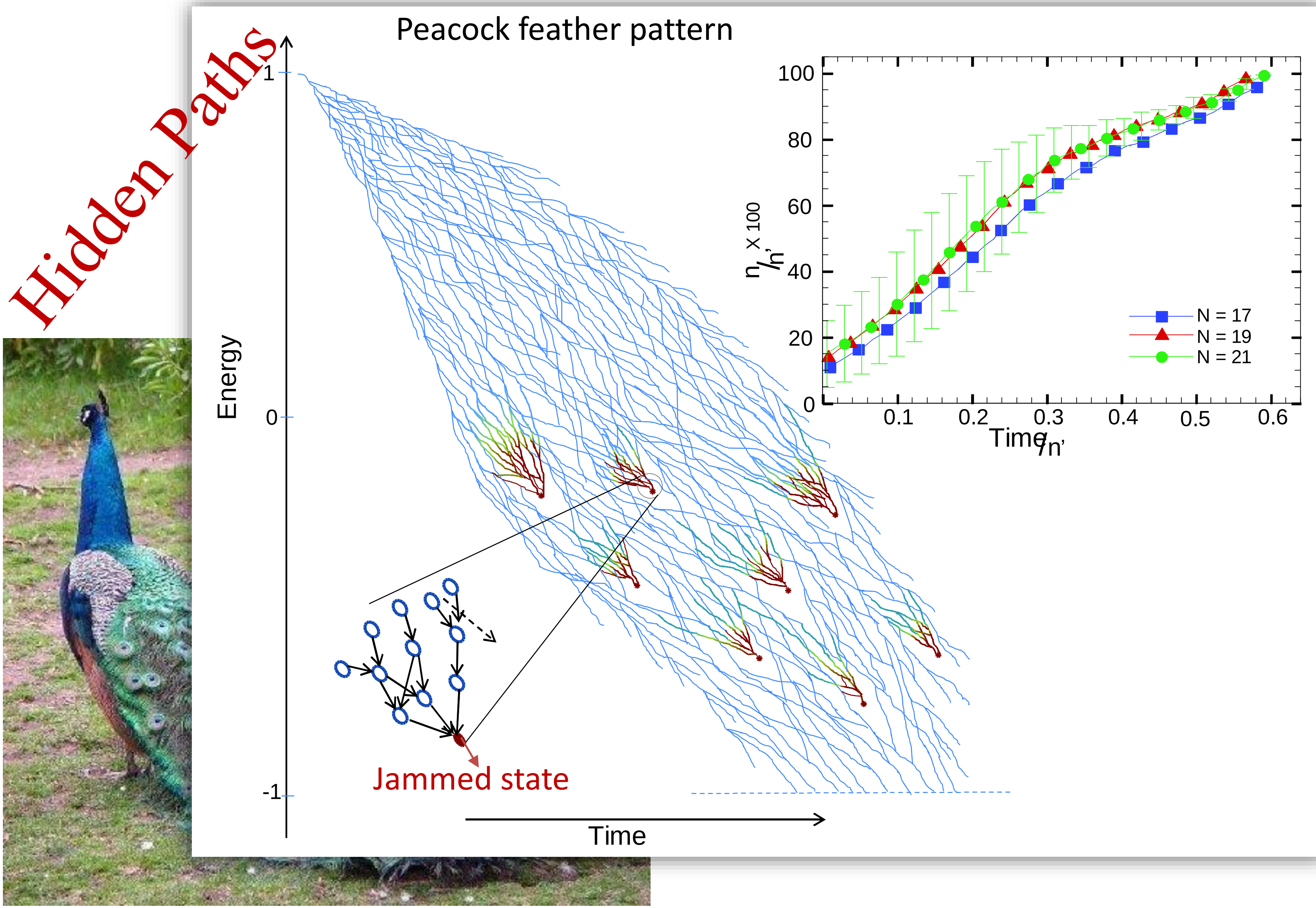
The **two definition** are **equivalent**:
An arbitrary signed graph is balanced under the first definition, if and only if, it is balanced under the second definitions

Balance test in incomplete networks



Note: Only negative edges among supernodes

Hidden Paths



Pseudo paths towards minimum energy states in network dynamics, L. Hedayatifar, F. Hassanibesheli, A.H. Shirazi, S.V. Farahani, G.R. Jafari, Physica A, 483, 109-116 (2017)

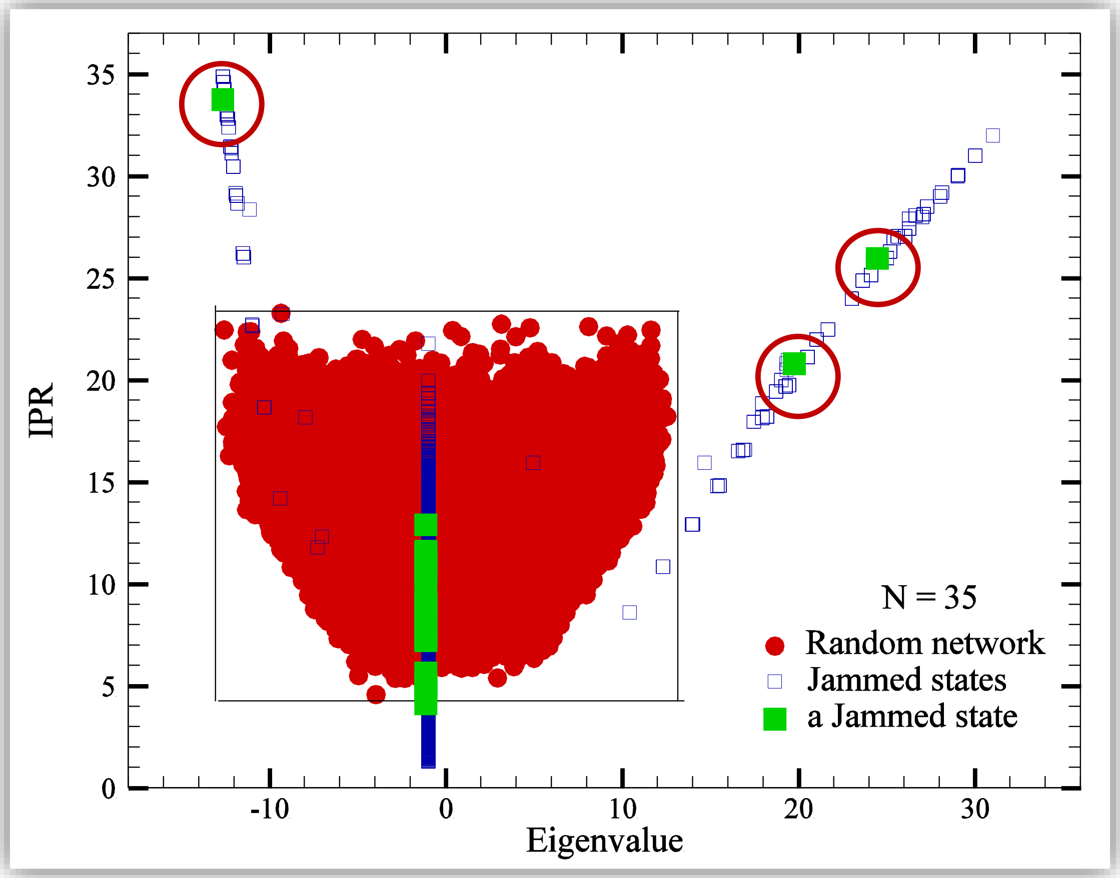
How can we detect or predict the Jammed states?

Inverse Participation Ratio

$$\mathrm{IPR} = \frac{1}{\sum_i |\psi_n(i)|^4}$$

$$|\psi> = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

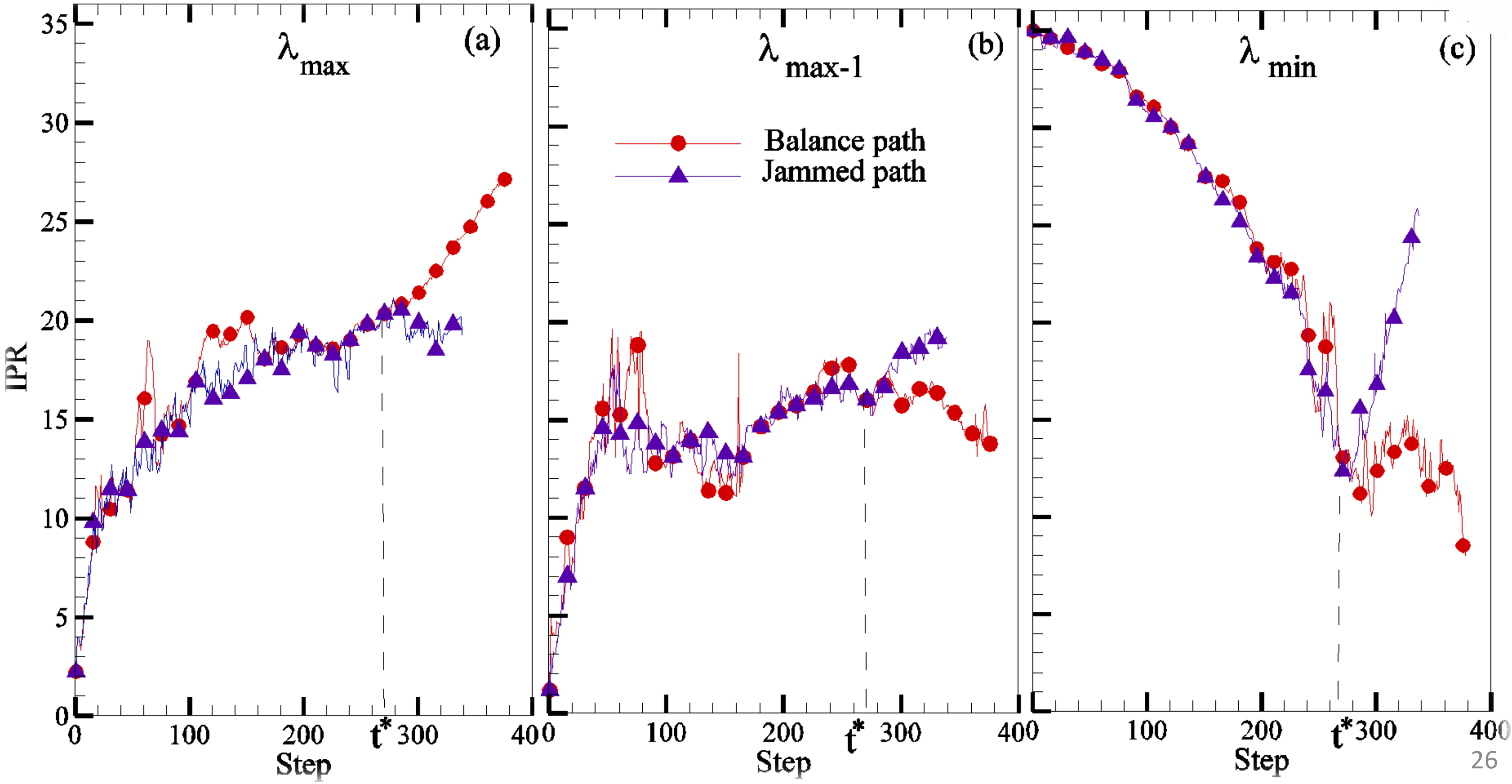
$$1 \leq \mathrm{IPR} \leq N$$



Inverse participation ratio (IPR)

$$\mathrm{IPR} = \frac{1}{\sum_i |\psi_n(i)|^4}$$

$1 \le \mathrm{IPR} \le N$



We do not forget our old friends/enemies easily

Our relationships are not mathematics parameters that we can change them easily.

Yesterdays' friend (enemy) rarely become tomorrows' enemy (friend).

Fractional calculus:

$$\frac{dx_{ki}}{dt} = \sum_{l=1} X_{kl}(t')X_{li}(t')$$

Fractional derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes.

$$\frac{dx_{ki}}{dt} = \int dt' K(t-t') \left[\sum_{l=1} X_{kl}(t')X_{li}(t')\right]$$

$0 < \alpha < 1$

$$\frac{dx_{ki}}{dt} = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} dt'(t-t')^{(\alpha-1)} \left[\sum_{l=1} X_{kl}(t')X_{li}(t')\right]$$

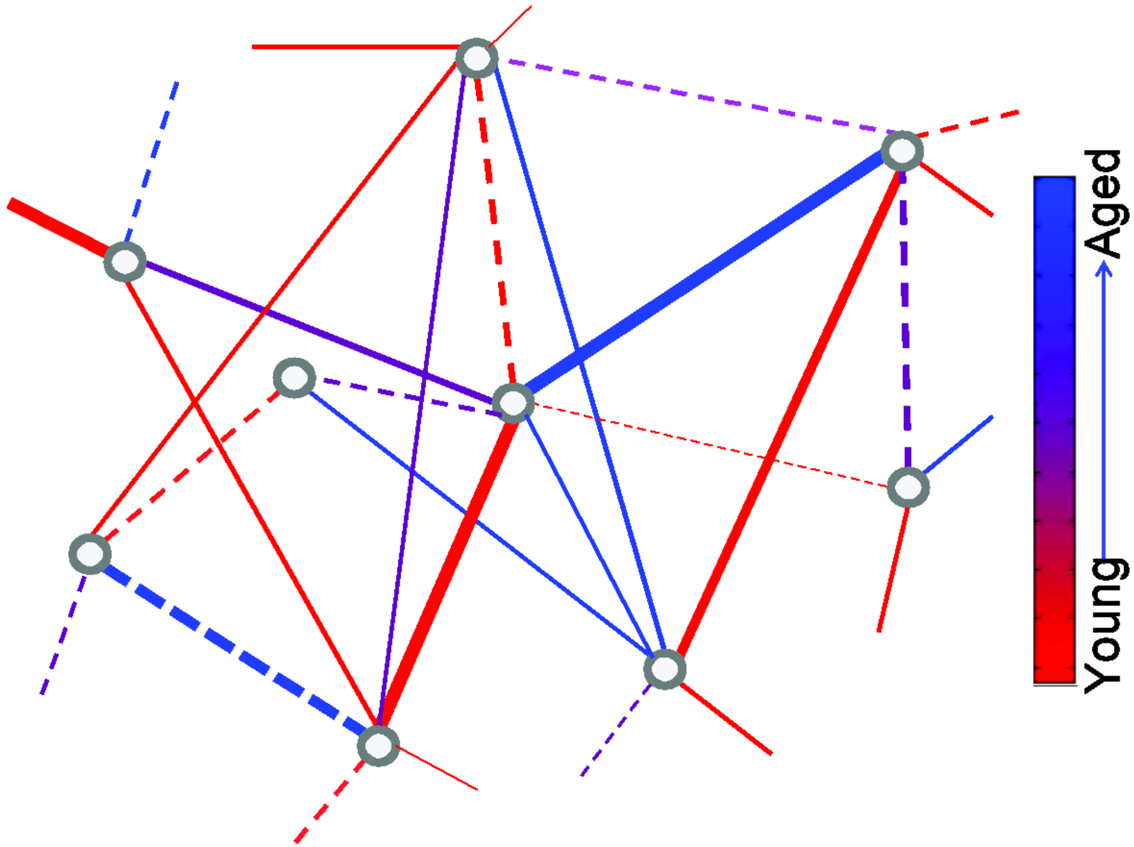
Caputo fractional differential operator

$$\frac{d^\alpha X_{ki}(t)}{dt^\alpha} = \sum_{j=1} X_{kj}(t)X_{ji}(t).$$

$$X_{ki} = X_{ki0} + h^\alpha \sum_{j=0}^{n-1} b_{n-j-1} \left[\sum_{l=1} (X_{kl}X_{li})_j\right]$$

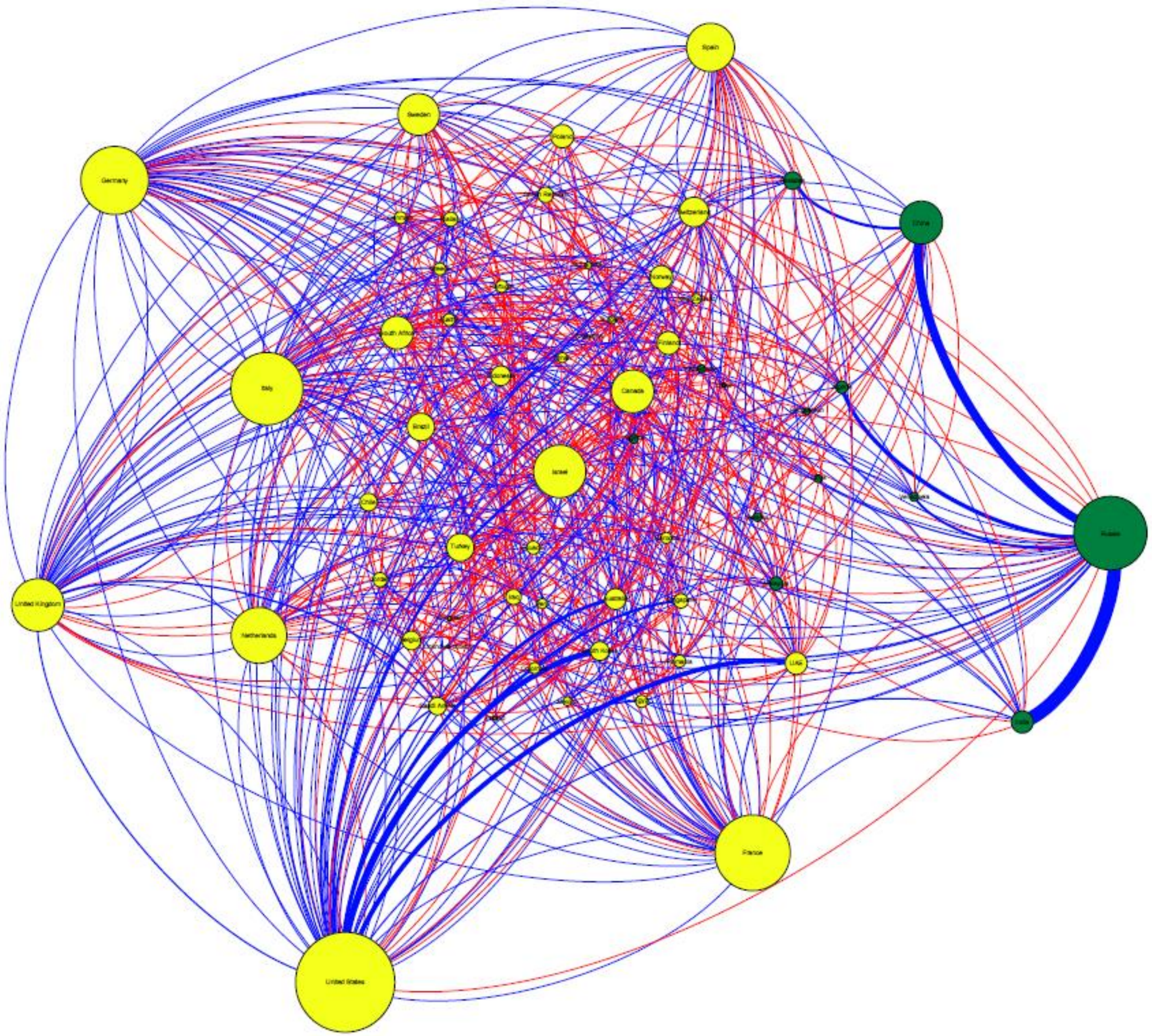
$$b_n = \frac{(n+1)^\alpha - (n)^\alpha}{\Gamma(\alpha+1)}$$

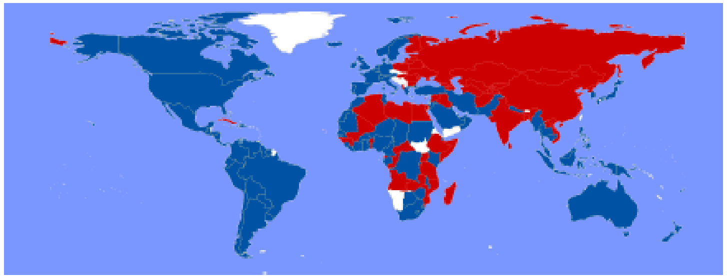
Aged Networks



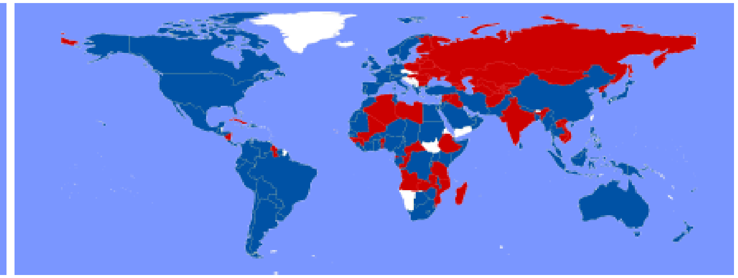
Glassy States of Aging Social Networks, F. Hassanibesheli, L. Hedayatifar, H. Safdari, M. Ausloos, G.R. Jafari, Entropy 19 (6), 246 (2017)
Fractional Dynamics of Network Growth Constrained by Aging Node Interactions, Plos One 0154983 (2016)

Weapon Trade Network

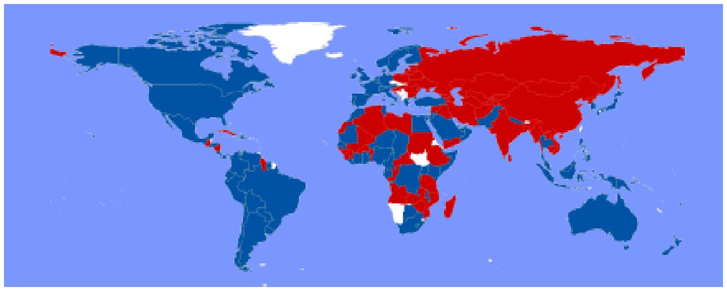




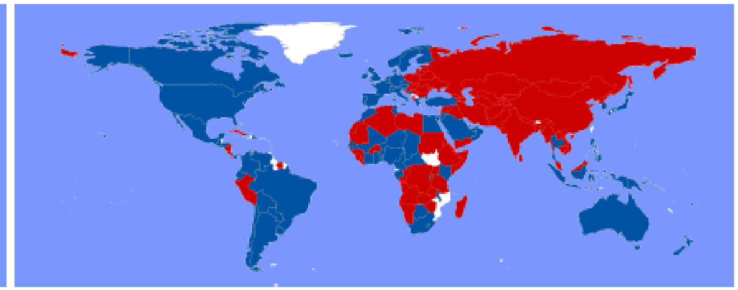
(c) 1971-1981



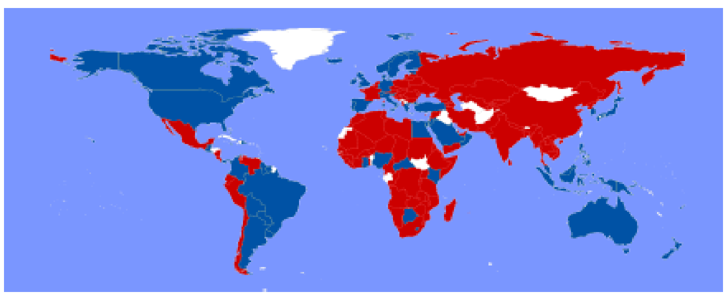
(d) 1976-1986



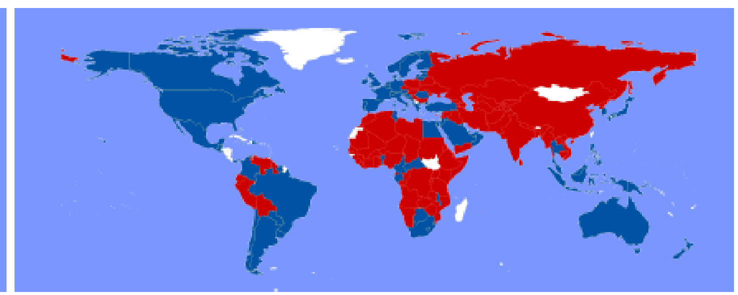
(e) 1981-1991



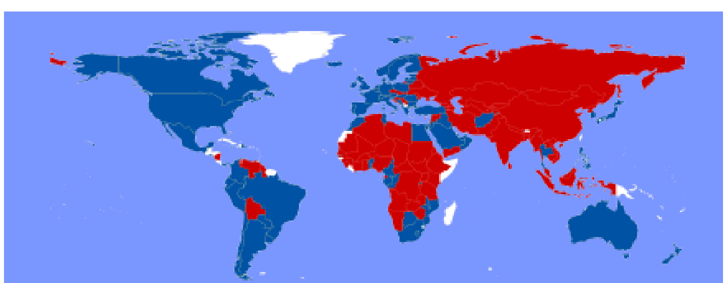
(f) 1986-1996



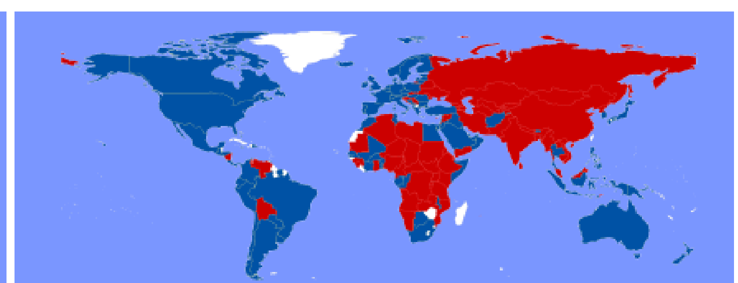
(g) 1991-2001



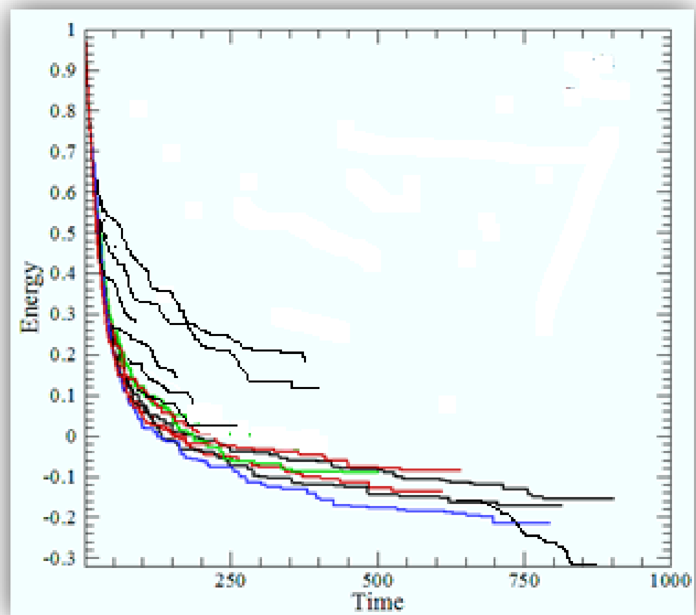
(h) 1996-2006



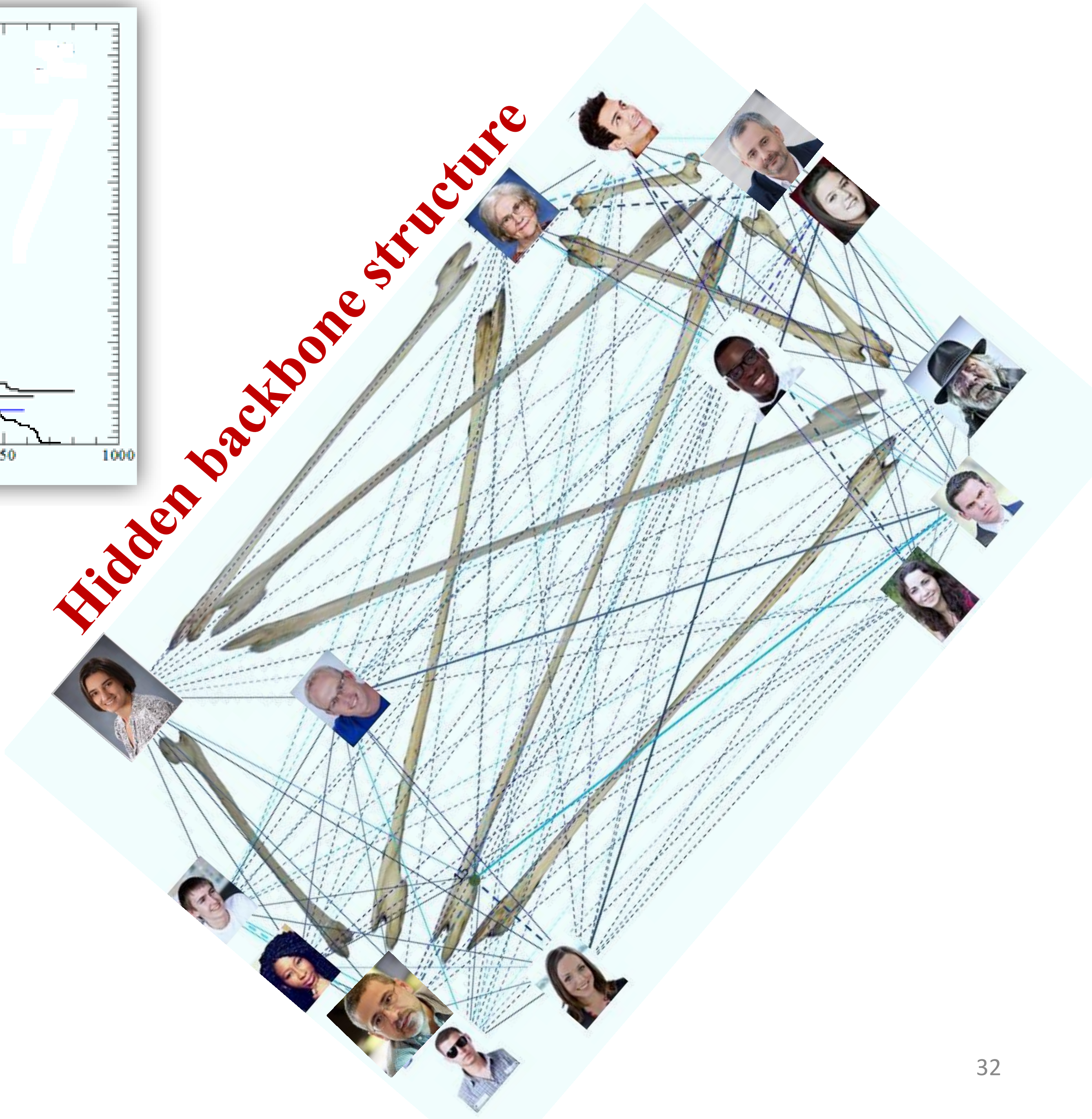
(i) 2001-2011



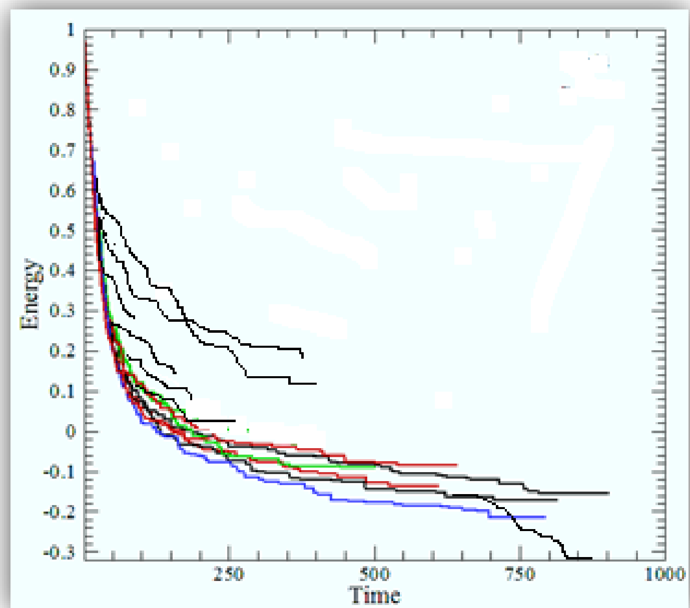
(j) 2006-2016

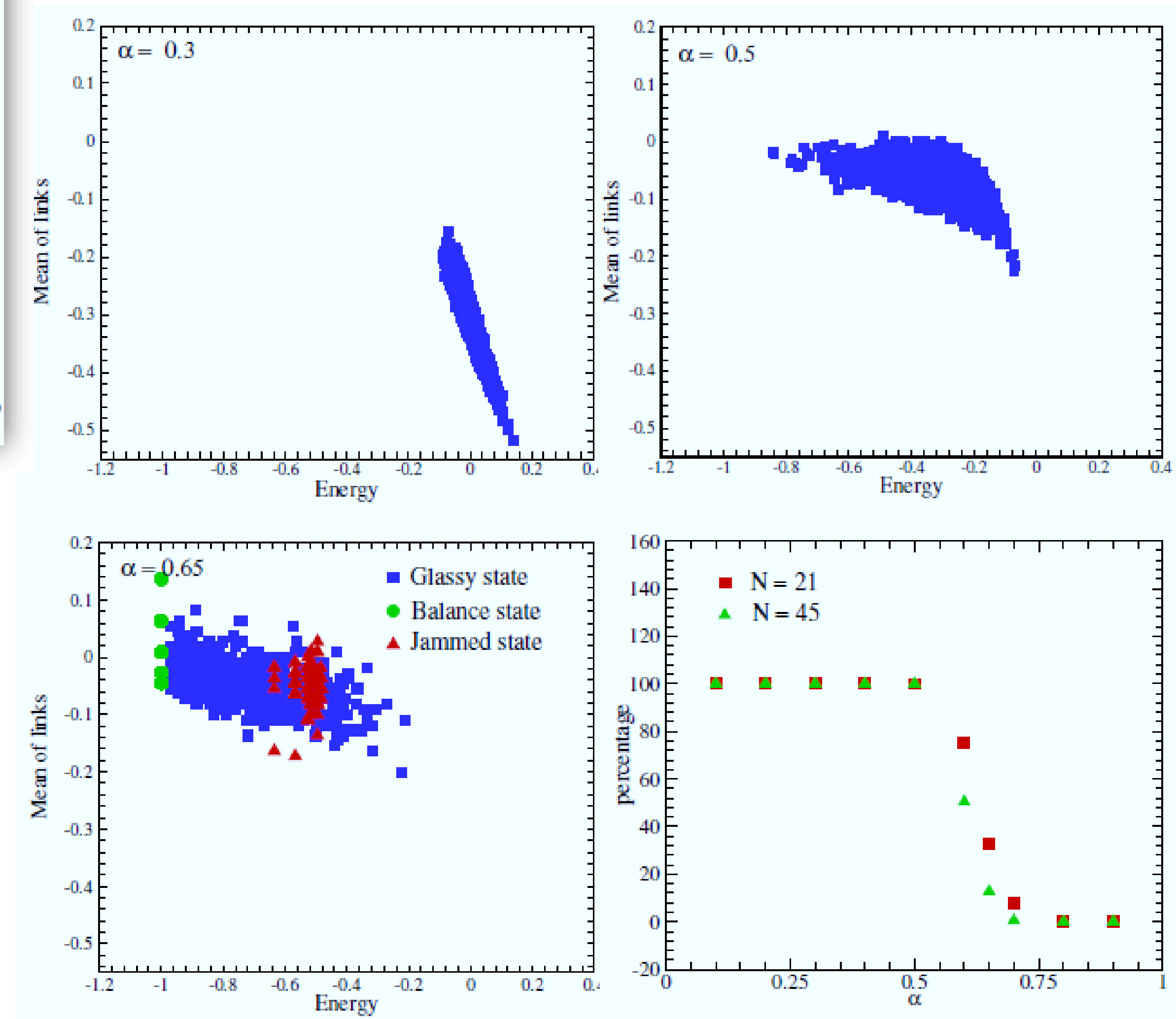


Hidden backbone structure



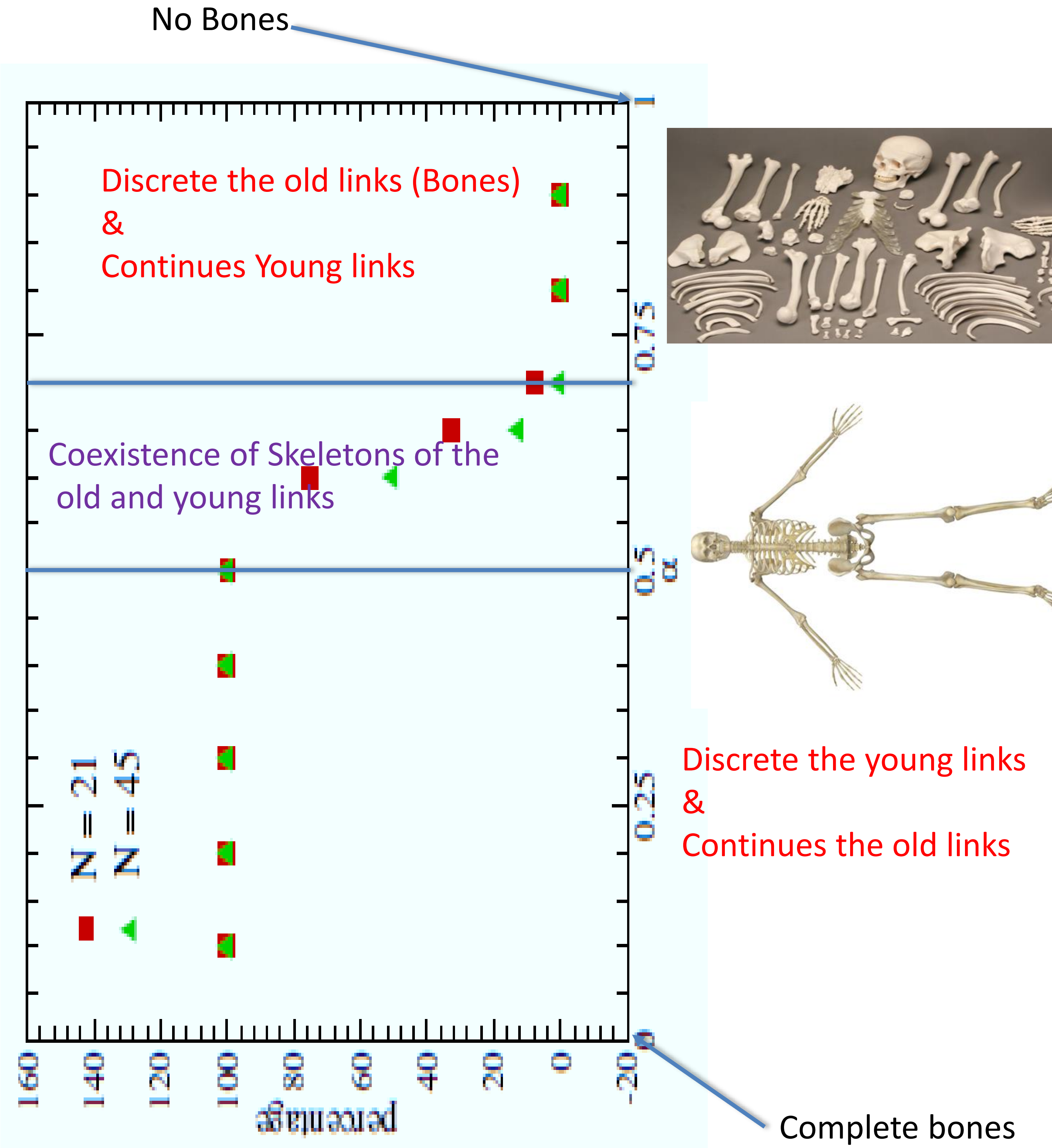
Glassy States





$$\frac{d^\alpha X_{ki}(t)}{dt^\alpha} = \sum_{j=1} X_{kj}(t) X_{ji}(t).$$

Depends on the value of α the amount of energy varies from negative to upper energy states even positive ones.

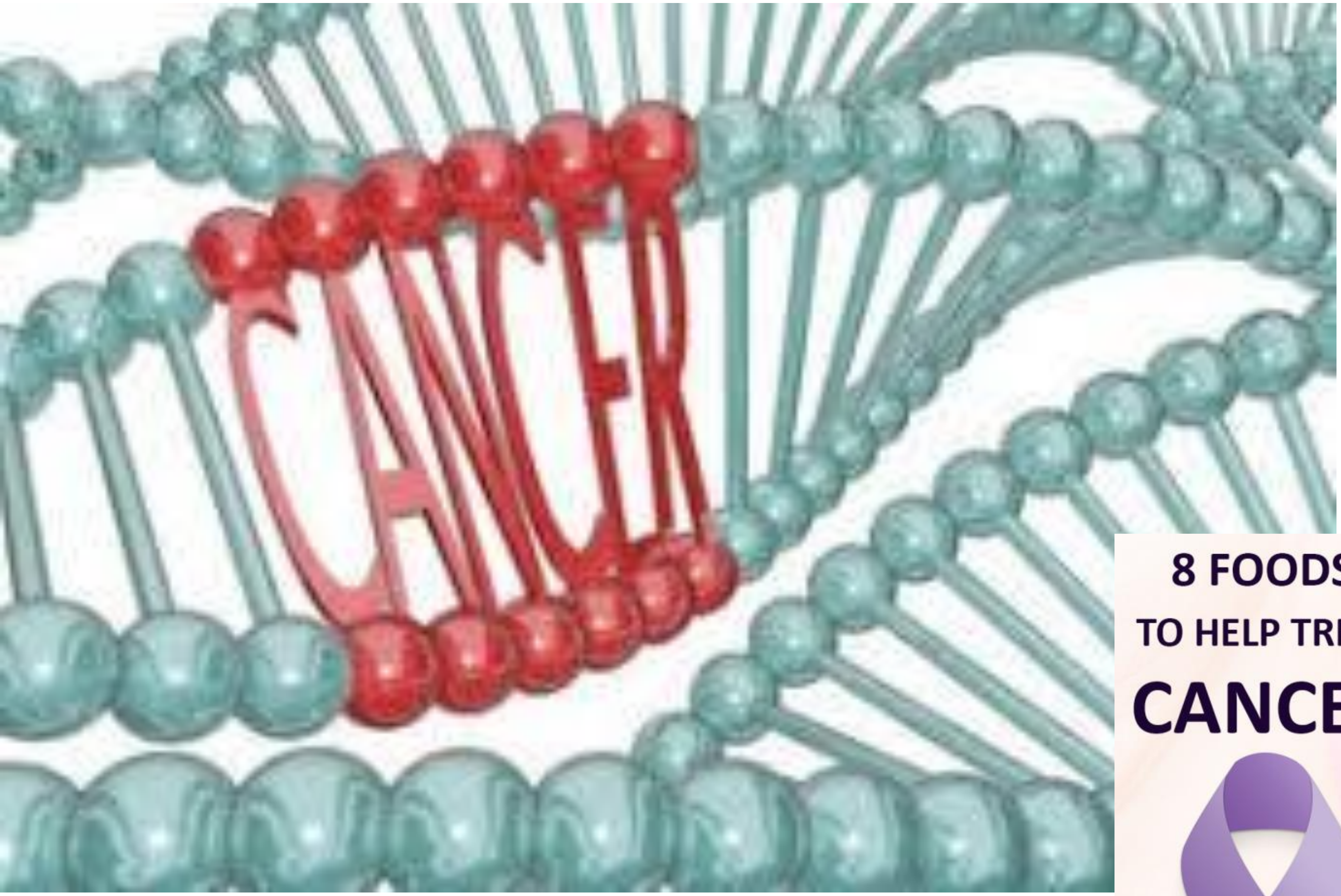


Bones – Skeleton phase transition

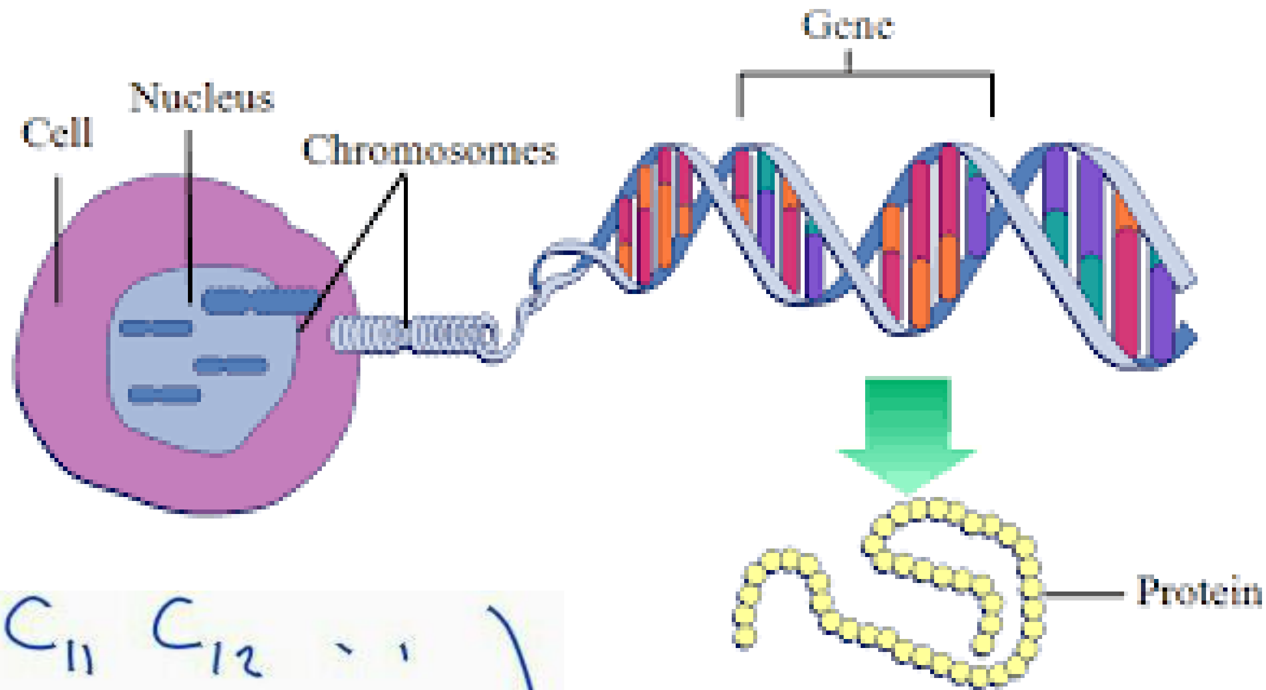
Two applications in **Cancer** and **Finance**

Cancer

Is known as a Gene disease.



Gene expression

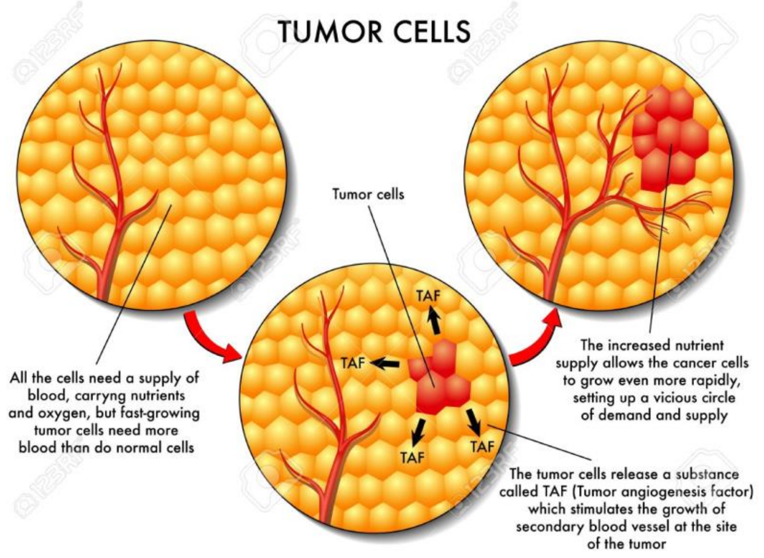


$$C = \begin{pmatrix} c_{11} & c_{12} & \cdots \\ c_{21} & c_{22} & \cdots \\ \vdots & & \ddots \\ \vdots & & \end{pmatrix}$$

Genomes society and Tumor

We used Prostate cancer data derived from NCBI Gene Omnibus GDS2545 dataset (on Human Genome platform).

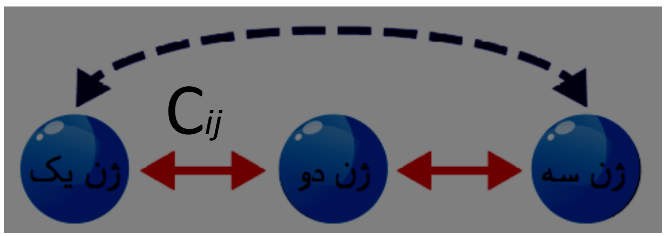
GDS2545 provides gene expression profiles of 171 samples (normal, normal adjacent to tumor, primary tumor, metastatic tumor).

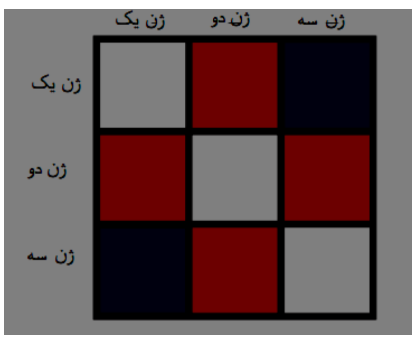


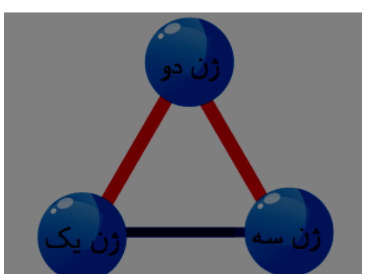
	Normal	Adjacent to tumor	Primary tumor	Metastatic tumor
#Samples	18	65	64	24

PARTIAL CORRELATION

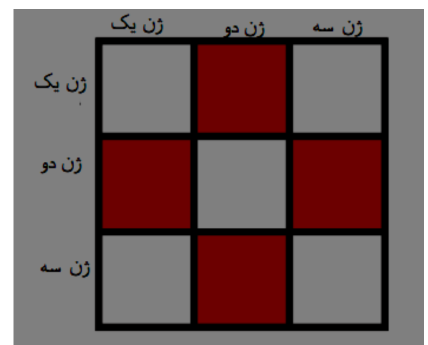
همبستگی

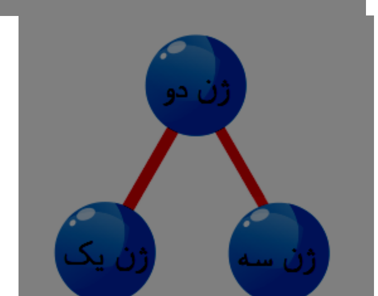




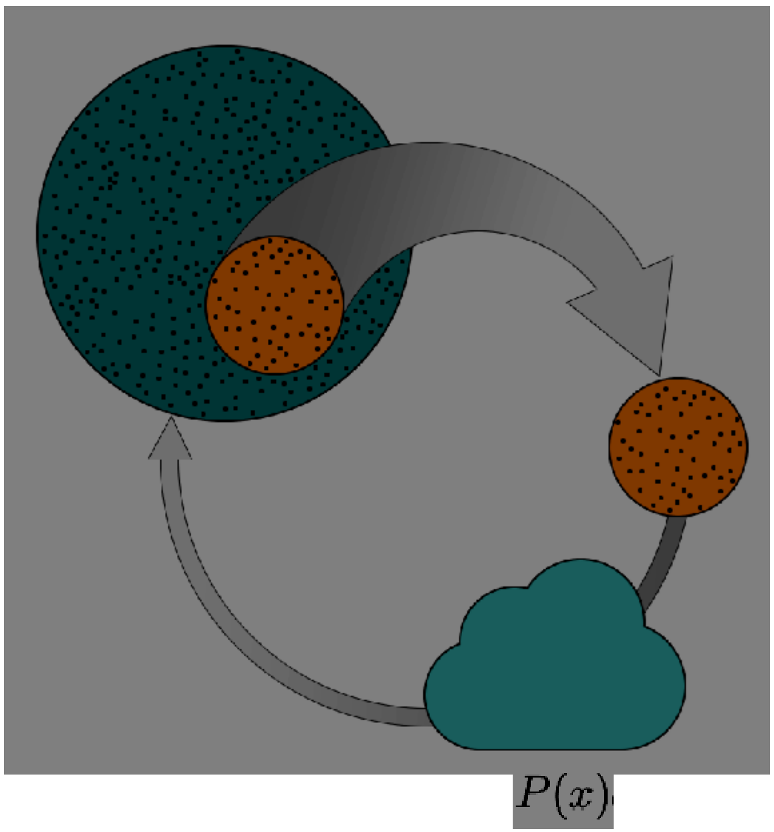


همبستگی جزئی





MAX ENTROPY



$$H = -\sum_{ij}^{N} J_{ij} s_i s_j - \sum_{i}^{N} h_i s_i$$

$$\int_x P(x) dx = 1$$

$$\langle x_i \rangle = \int_x P(x) x_i dx = \frac{1}{M} \sum_{m=1}^{M} x_i^m = \overline{x_i}$$

$$\langle x_i x_j \rangle = \int_x P(x) x_i x_j dx = \frac{1}{M} \sum_{m=1}^{M} x_i^m x_j^m = \overline{x_i x_j}$$

$$maximize S = -\int_x P(x)\ln P(x)dx$$

$$L = L(P(x);\alpha,\beta,\gamma)$$

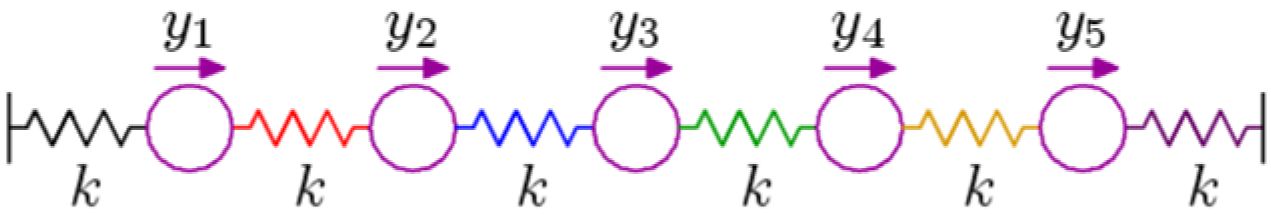
$$\frac{\delta L}{\delta P(x)} = ۰$$

$$P(x,\beta,\gamma) = \exp(-۱+\alpha+\sum_{i}^{L}\beta_i x_i + \sum_{i,j}^{L}\gamma_{ij}x_i x_j) = \frac{۱}{Z}e^{-H(x,\beta,\gamma)}$$

$$P(x;<x>,C) = (۲\pi)^{\frac{-L}{۲}} det(C)^{\frac{-۱}{۲}} \exp(-\frac{۱}{۲}(x-<x>)^T C^{-۱}(x-<x>))$$

$$J_{ij} = -\frac{1}{2}C^{-1}$$

Inverse-covariance matrix



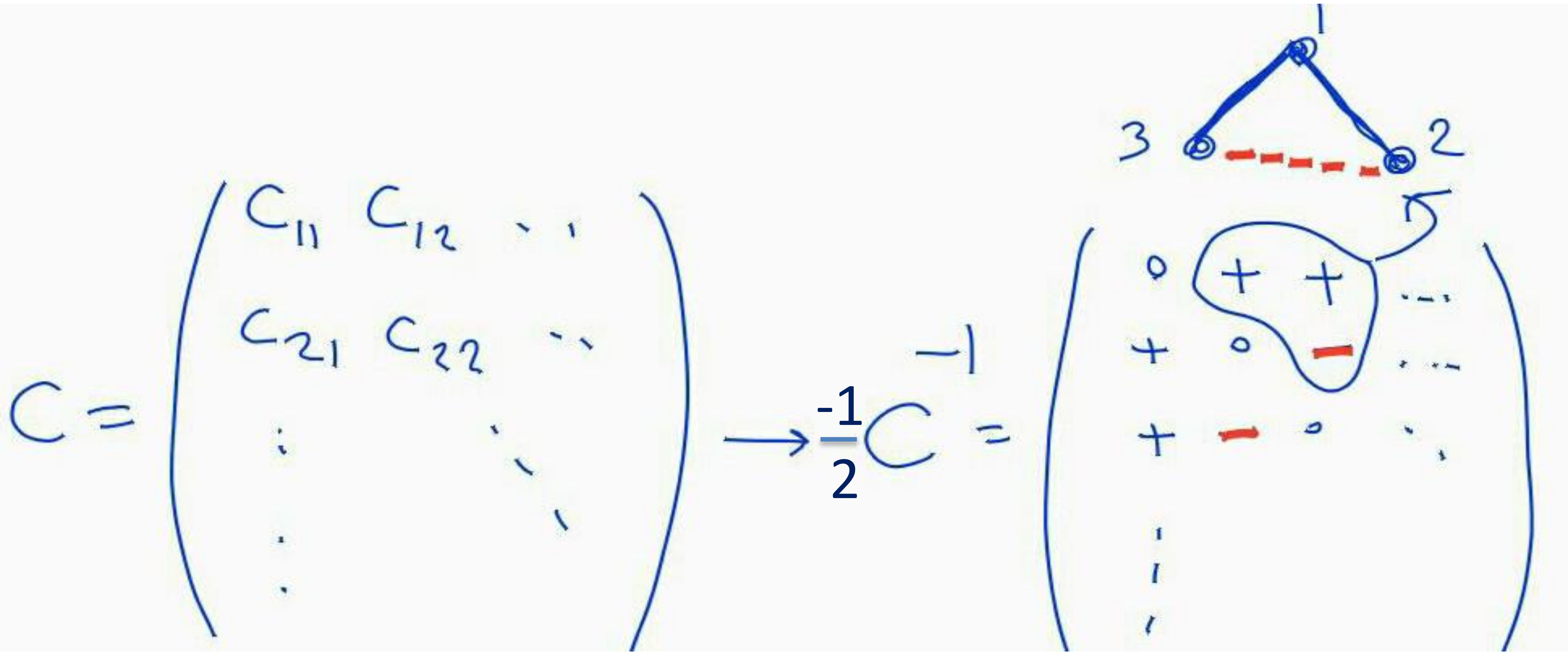
inverse-covariance matrix or covariance matrix?

$$\mathbf{K}^{-1} = \frac{k}{T}\begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\mathbf{K} = \frac{T}{k}\begin{bmatrix} 1.00 & 0.67 & 0.50 & 0.33 & 0.17 \\ 0.67 & 1.00 & 1.00 & 0.67 & 0.33 \\ 0.50 & 1.00 & 1.00 & 1.00 & 0.50 \\ 0.33 & 0.67 & 1.00 & 1.00 & 0.67 \\ 0.17 & 0.33 & 0.50 & 0.67 & 1.00 \end{bmatrix}$$

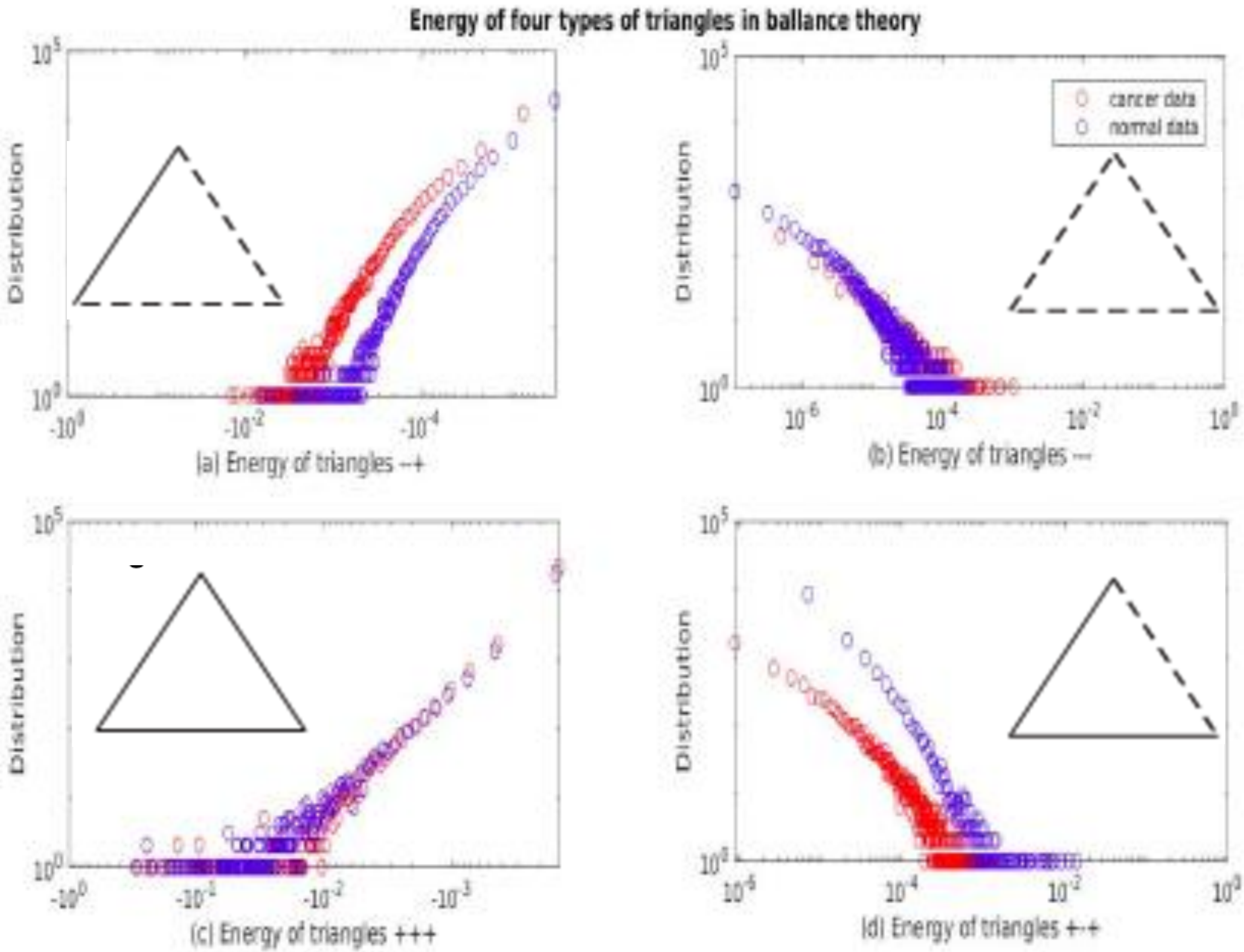
Correlation matrix between Genes activity
&
Interaction matrix

$$H = -\frac{1}{\binom{n}{3}} \sum s_{ij} s_{jk} s_{ik}$$

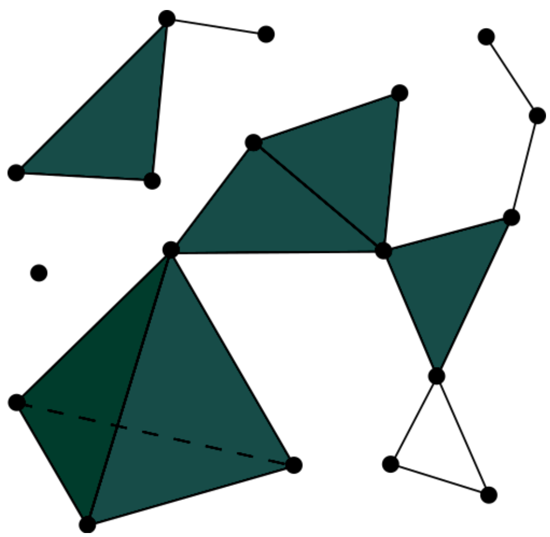


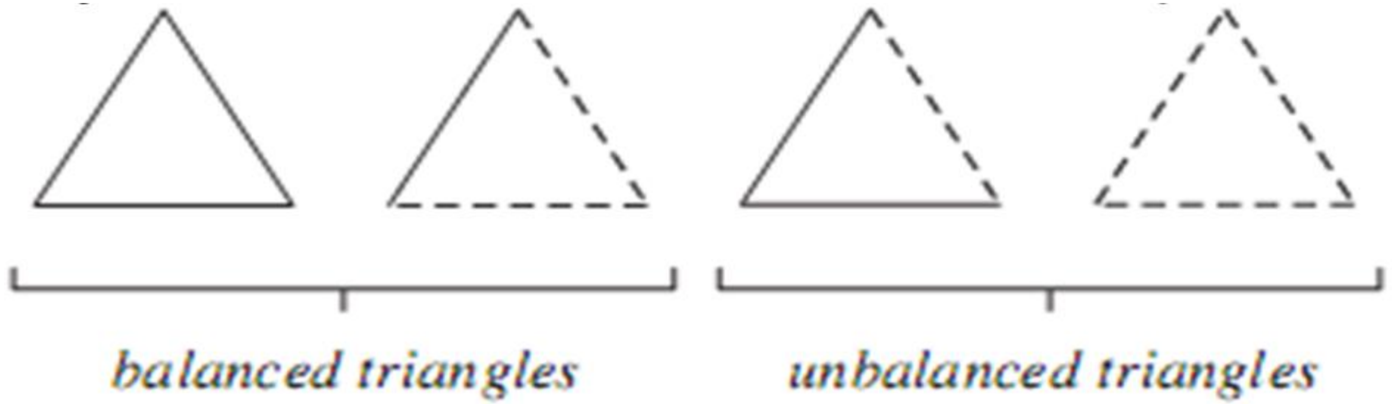
Lezon TR, Banavar JR, Cieplak M, Maritan A, Fedoroff NV, PNAS, 12;103(50):19033-8 (2006)

Energy distribution



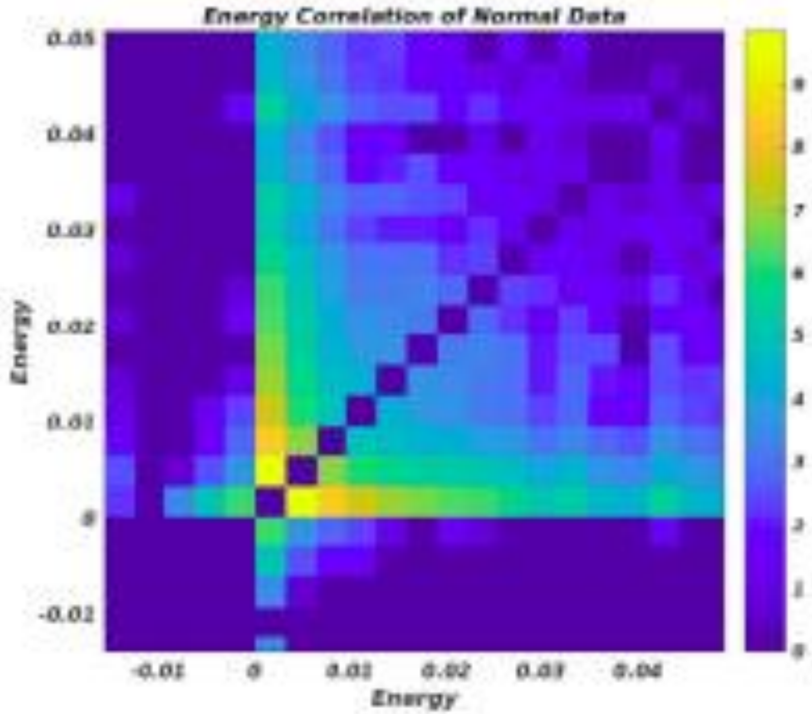
Energy-Energy Correlation



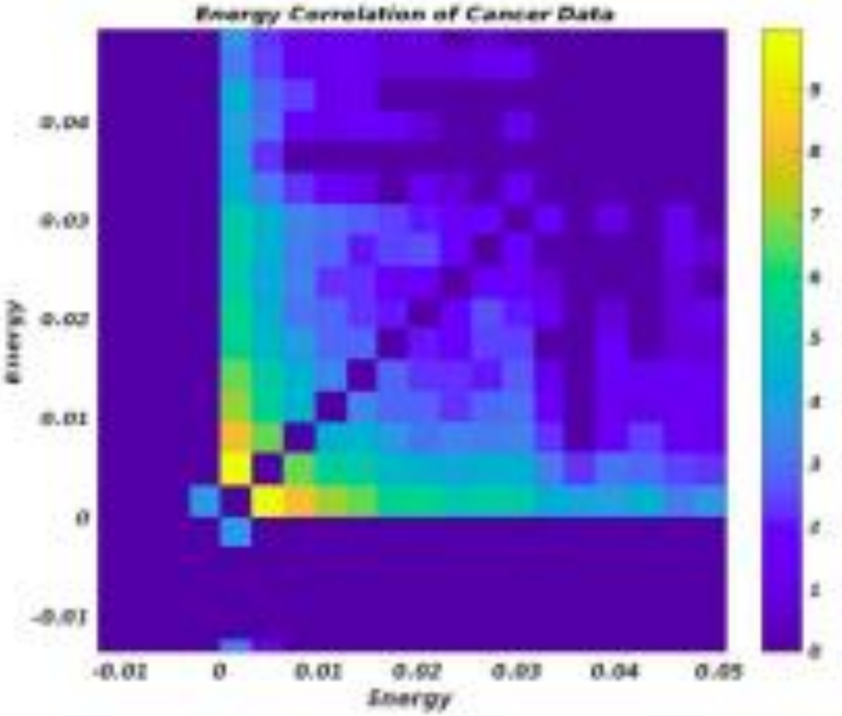


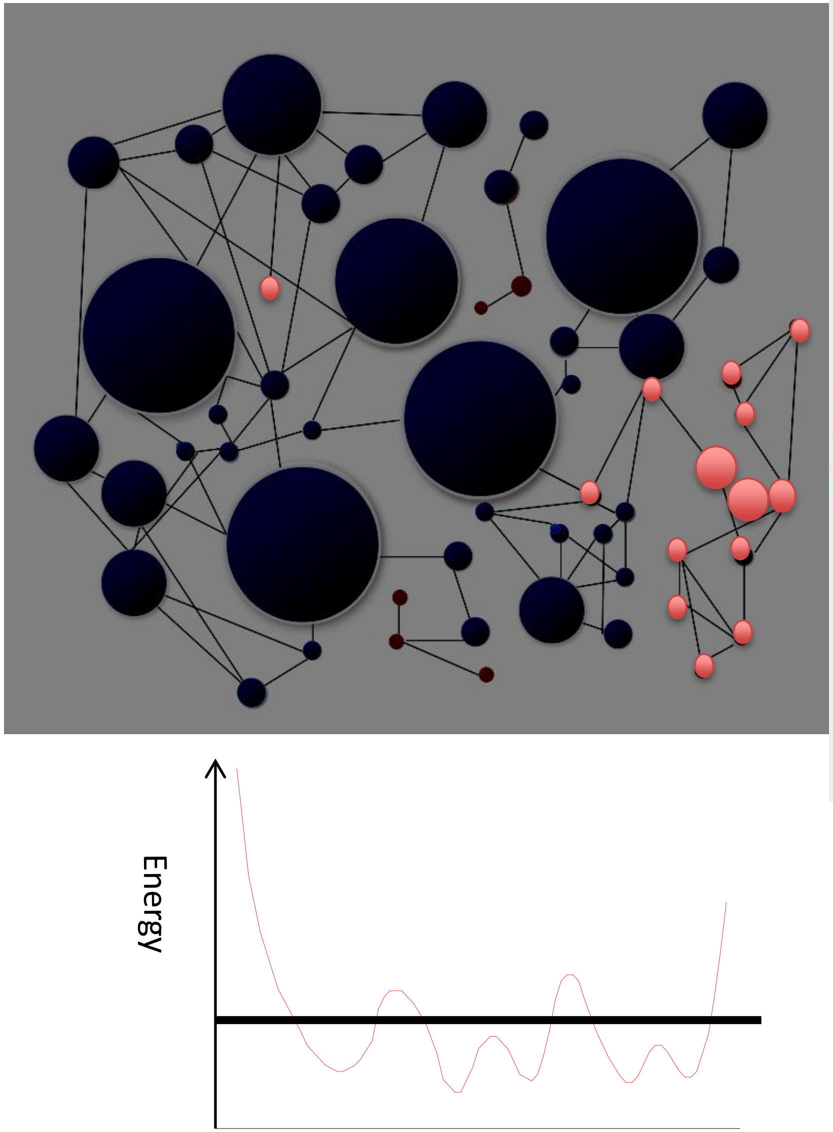
Energy – Energy correlation

Normal

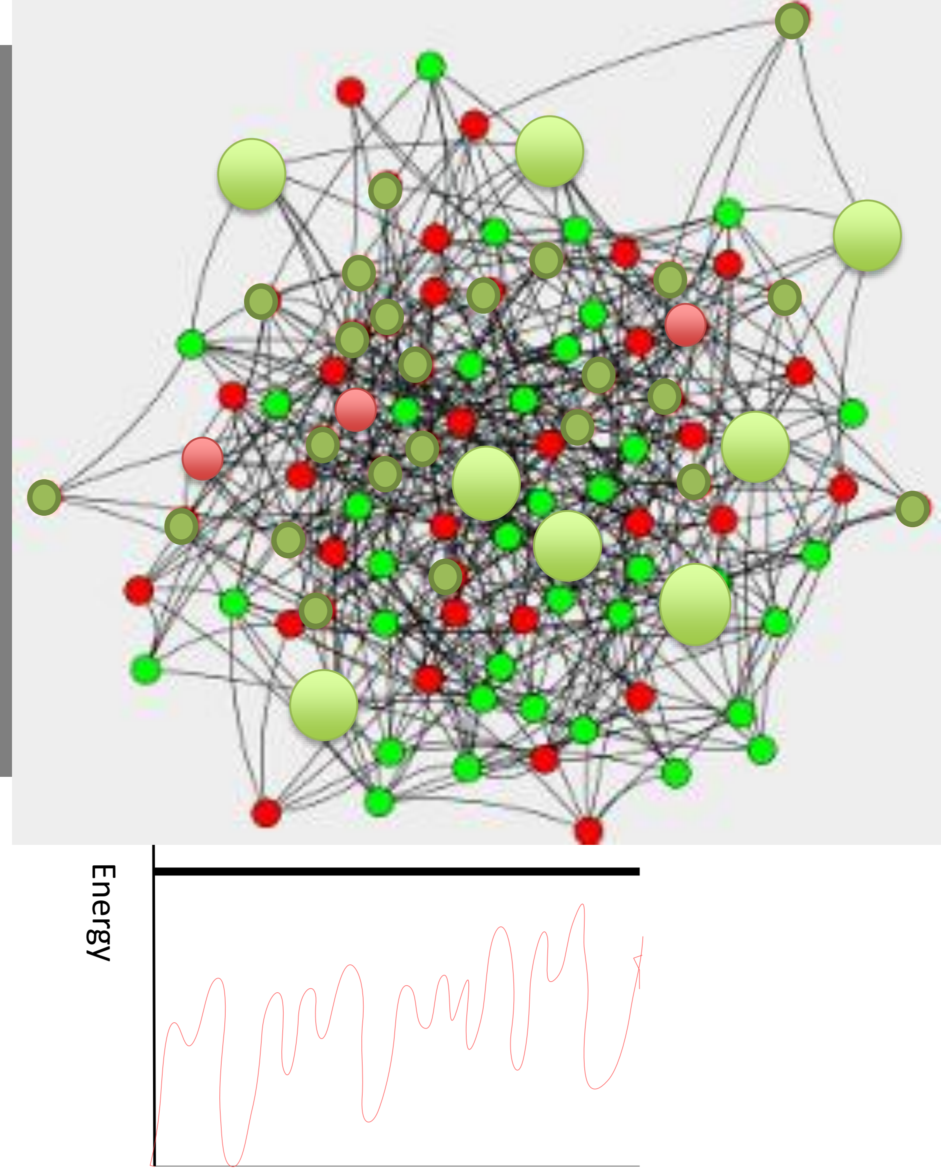


Cancer





Cancer



Normal

$\lambda_1 \qquad \lambda_2 \qquad \lambda_N$

$$|\psi_1\rangle = \begin{pmatrix}\alpha_1^1\\ \alpha_2^1\\ \vdots \\ \alpha_N^1\end{pmatrix}, \quad |\psi\rangle = \begin{pmatrix}\alpha_1^2\\ \alpha_2^2\\ \vdots \\ \alpha_N^2\end{pmatrix} \quad \cdots\cdots$$

$$NPR_1 = \sum_{i=1}^{N} |\alpha_1^i|^4$$

$$IPR_1 = \frac{1}{\sum_{i=1}^{N} |\alpha_i^1|^4} \quad \text{- - - - -}$$

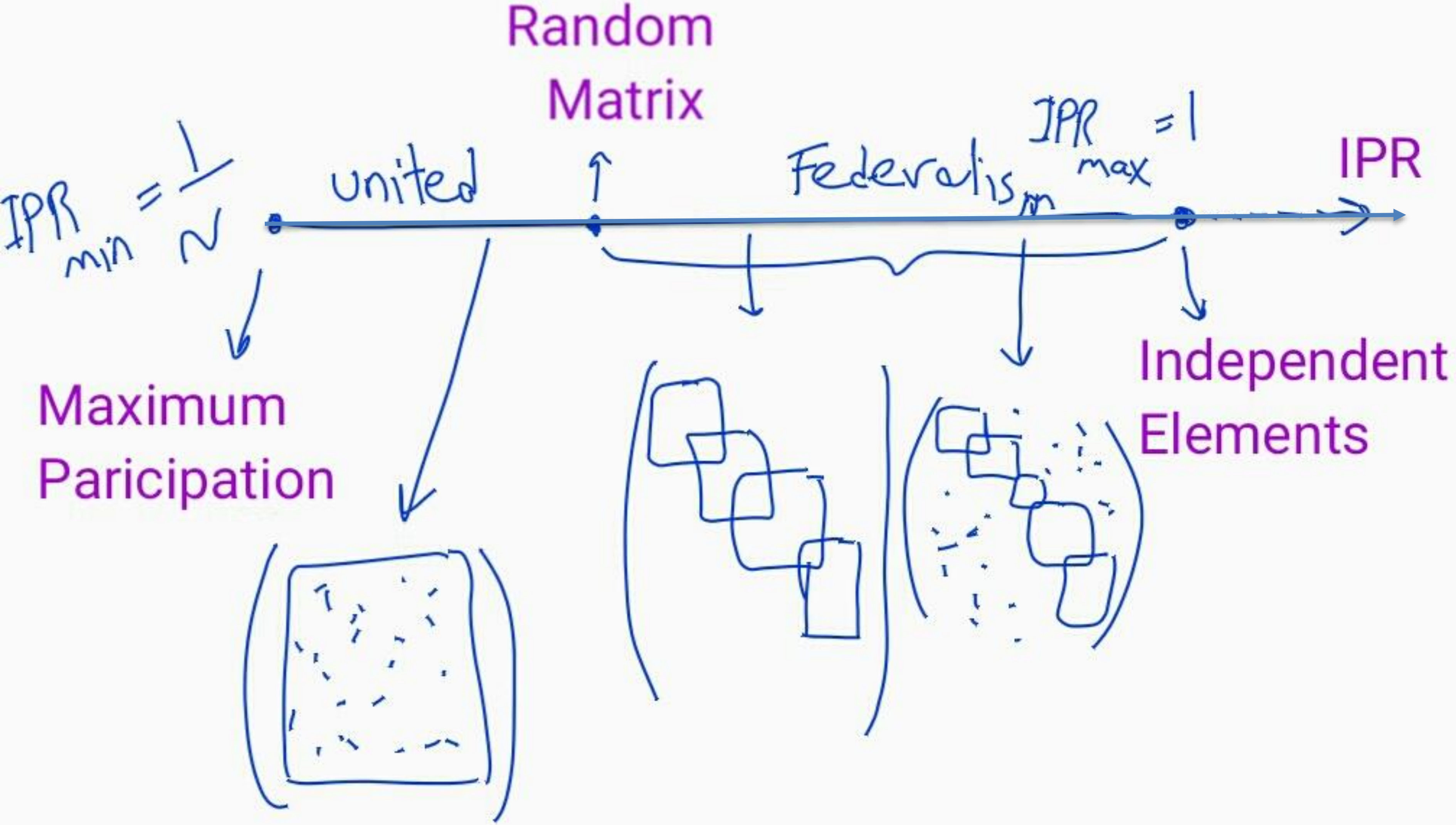
$$|\psi\rangle = \begin{pmatrix}0\\ \vdots \\ 1 \\ \vdots \\ 0\end{pmatrix}$$

$$IPR_{max} = 1$$

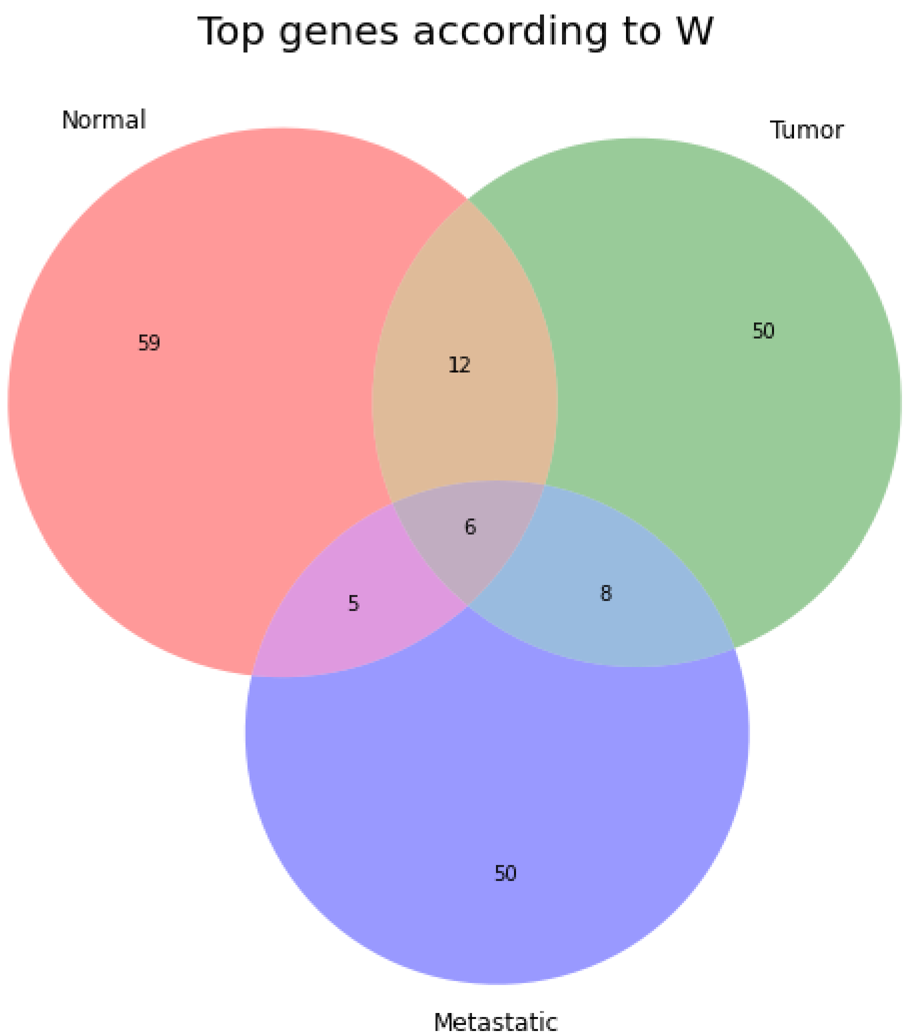
$$|\psi\rangle = \frac{1}{\sqrt{N}}\begin{pmatrix}1\\ \vdots \\ 1\end{pmatrix}$$

$$IPR_{min} = \frac{1}{\sum_{i=1}^{N} \left|\frac{1}{\sqrt{N}}\right|^4} = \frac{1}{N}$$

$$PR_1 = \sum_{i=1}^{N} |\alpha_i^1|^4$$



Random
Matrix
$IPR_{min} = \frac{1}{N}$
united
Federalism
$IPR_{max} = 1$
IPR
Maximum
Paricipation
Independent
Elements

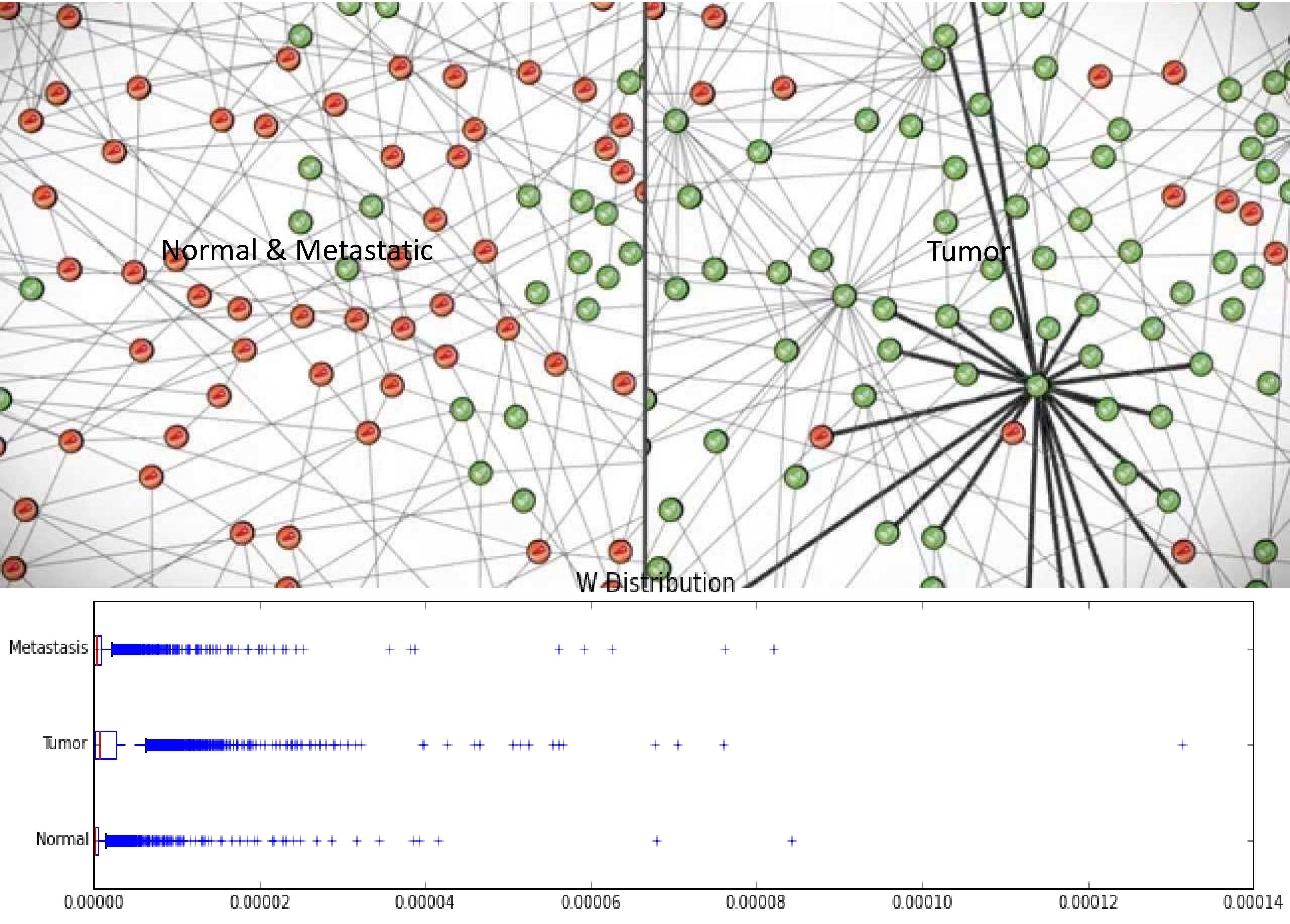

Top genes according to W
Normal
Tumor
59
12
50
6
5
8
50
Metastatic

Top driver genes in metastatic cells

ALB	M64936	CSTA	IGL@	APOA1	DST	LOC10012658 3	D26561	ESM1	NF1
PCK1	RBP4	MUC1	FN1	KLK3	CYP3A7	HPR	ACTG2	PDE5A	ALDOB
CRP	IGL@	CHI3L1	FN1	FABP4	ALB	KLK2	IRS1	MAPK8	TGM4
MYH11	IGHV4-31	MCF2	UGT2B15	PSPH	FGG	HPGD	APOB	HPGD	STAC
KLK3	GATM	SULT1C2	PRKG2	MAGEB1	PRG4	SSX2B	KLK3	FGB	SLC25A13
FGA	KLK2	FABP1	DDC	C5	MHY11	CPB1	PLAT	IL2	PF4V1
IGK@	SLC25A24	MSMB	AF070543	ORM2	IGH@	KNG1	PCK1	SERPINA1	

Cancer related
Immune system related
Growth related

Adhesion and migration related
Proliferation and differentiation


Normal & Metastatic
Tumor
W Distribution
Metastasis
Tumor
Normal
0.00000
0.00002
0.00004
0.00006
0.00008
0.00010
0.00012
0.00014

Application on Stock markets

Covariance Matrix
of
log-return price of each company

S&P500 Crisis

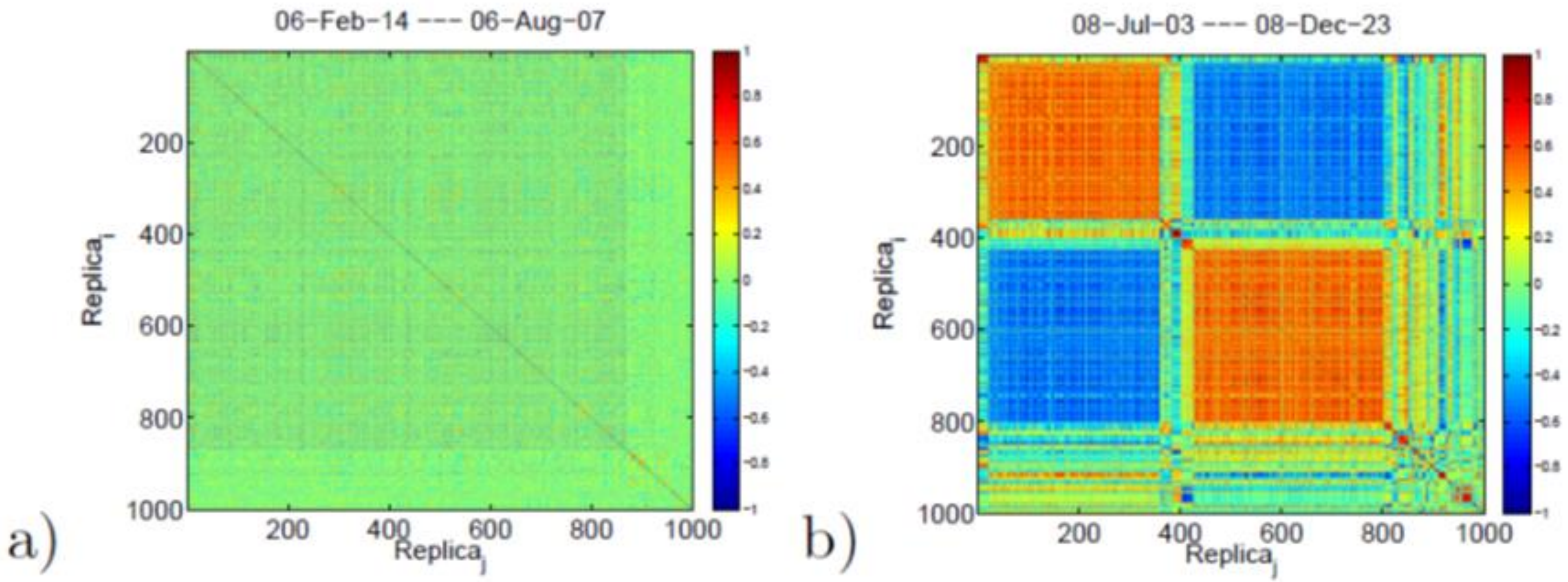
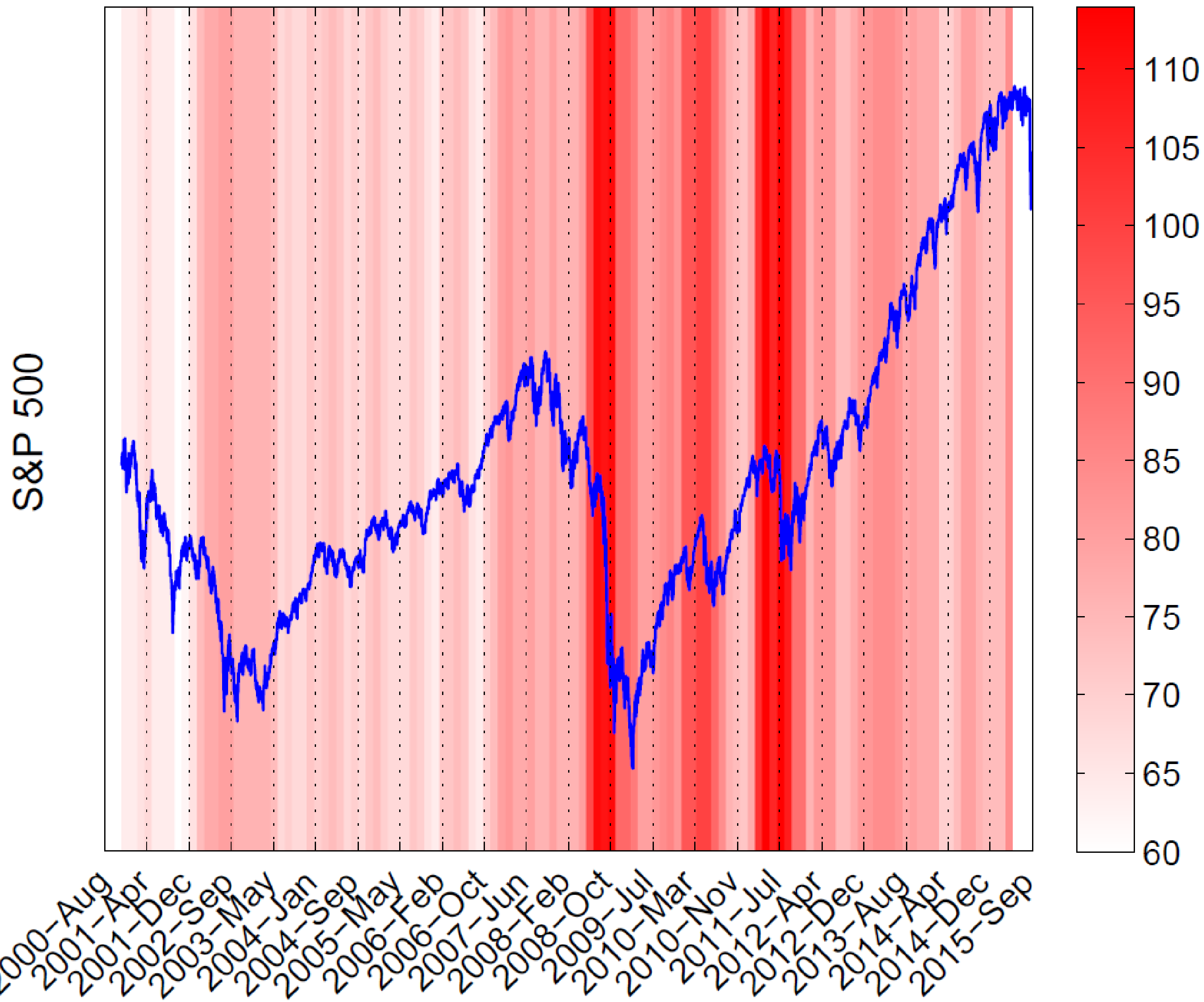


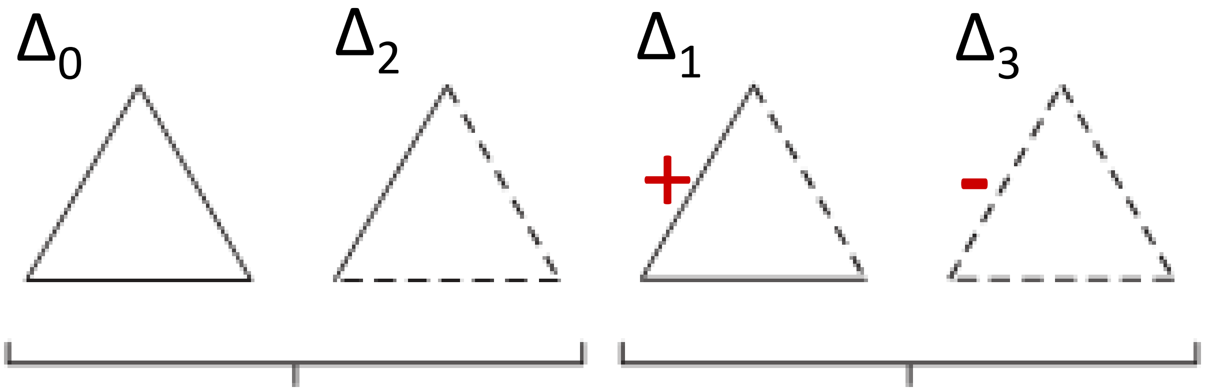
FIG. 2. a) Crisis and b) off-crisis replica correlation matrix Rows and columns were rearranged to demonstrate groups and blocks of replicas

Heat map of S&P500



Structural balance

$$H = -\frac{1}{\binom{n}{3}} \sum s_{ij} s_{jk} s_{ik}$$



balanced triangles

$S_{ij}\ S_{jk}\ S_{ki}=1$

Even number of dashed edges

unbalanced triangles

$S_{ij}\ S_{jk}\ S_{ki}=-1$

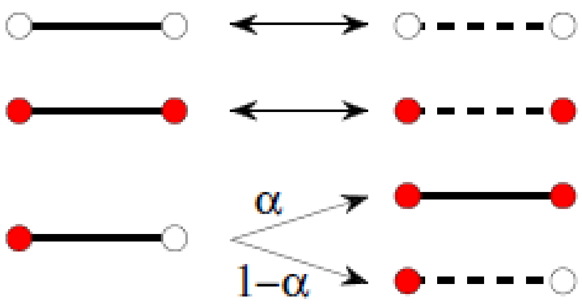
Odd number of dashed edges

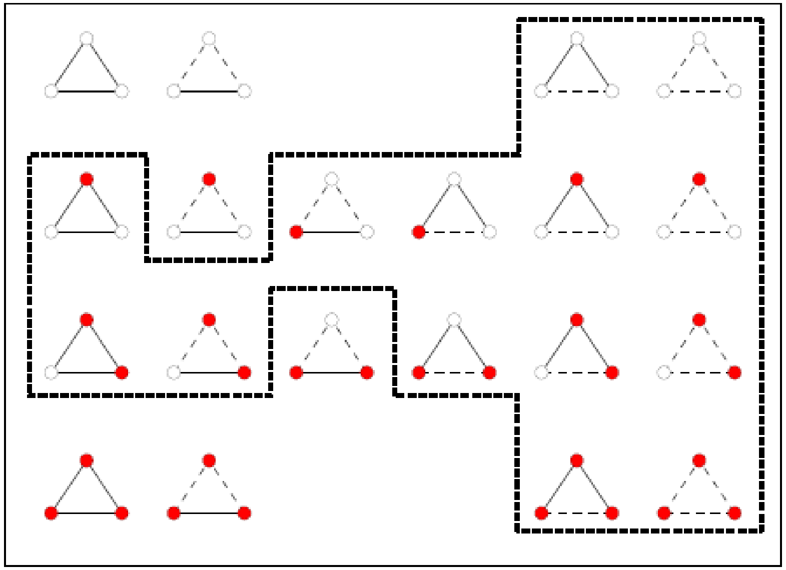
Δ_2: An enemy of my friend is my enemy

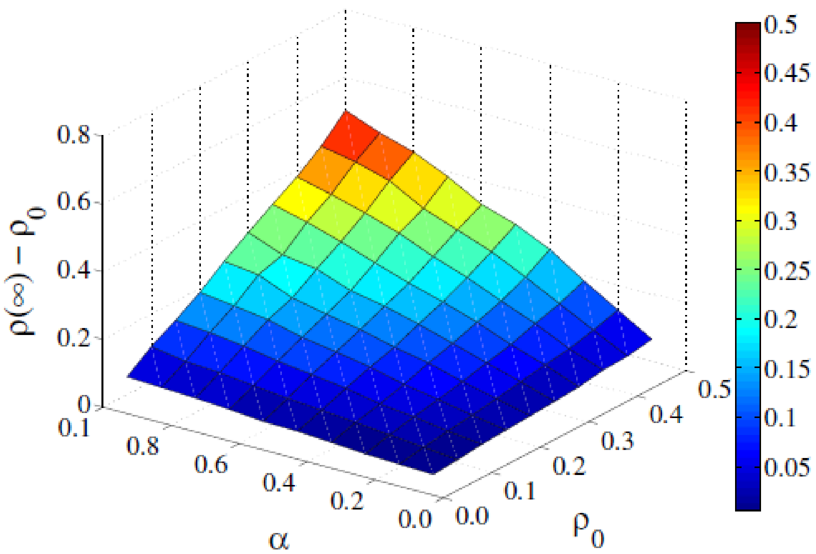
Δ_3: An enemy of my enemy is my enemy

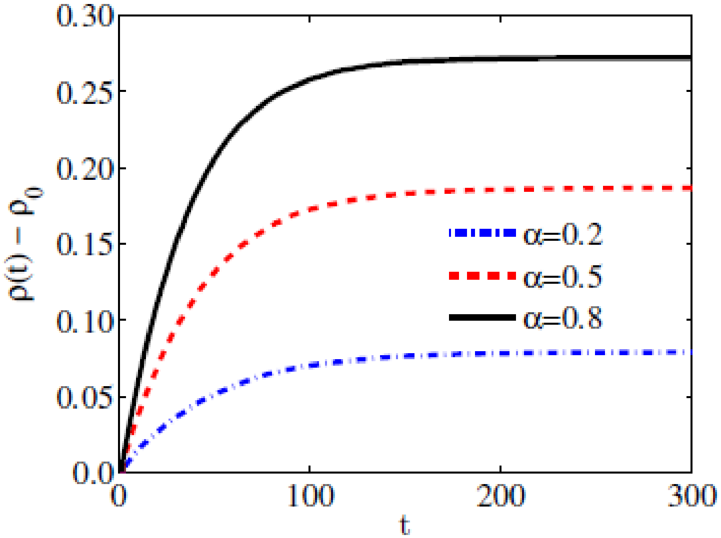
T. Antal, P. L. Krapivsky, and S. Redner, Phy. Rev. E 72. 036121 (2005)

Epidemic process in the evolving network







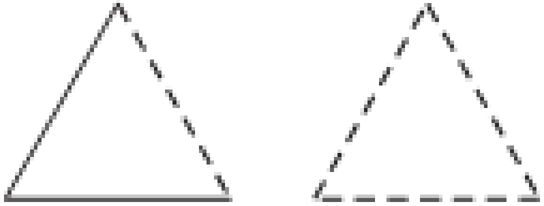


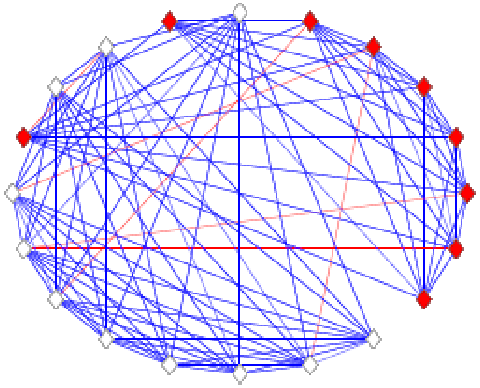
Initial conditional 30% Infected

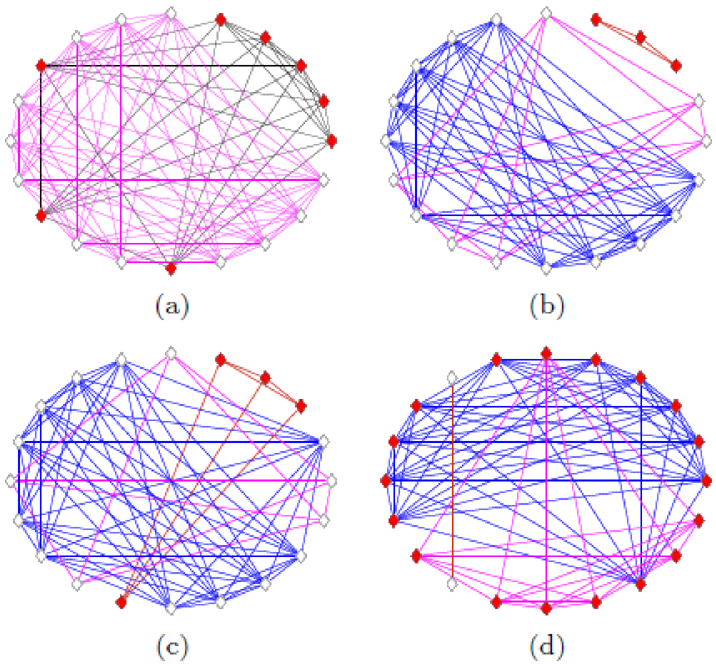
Initial conditions and Jammed states

Fully un-friendship Networks

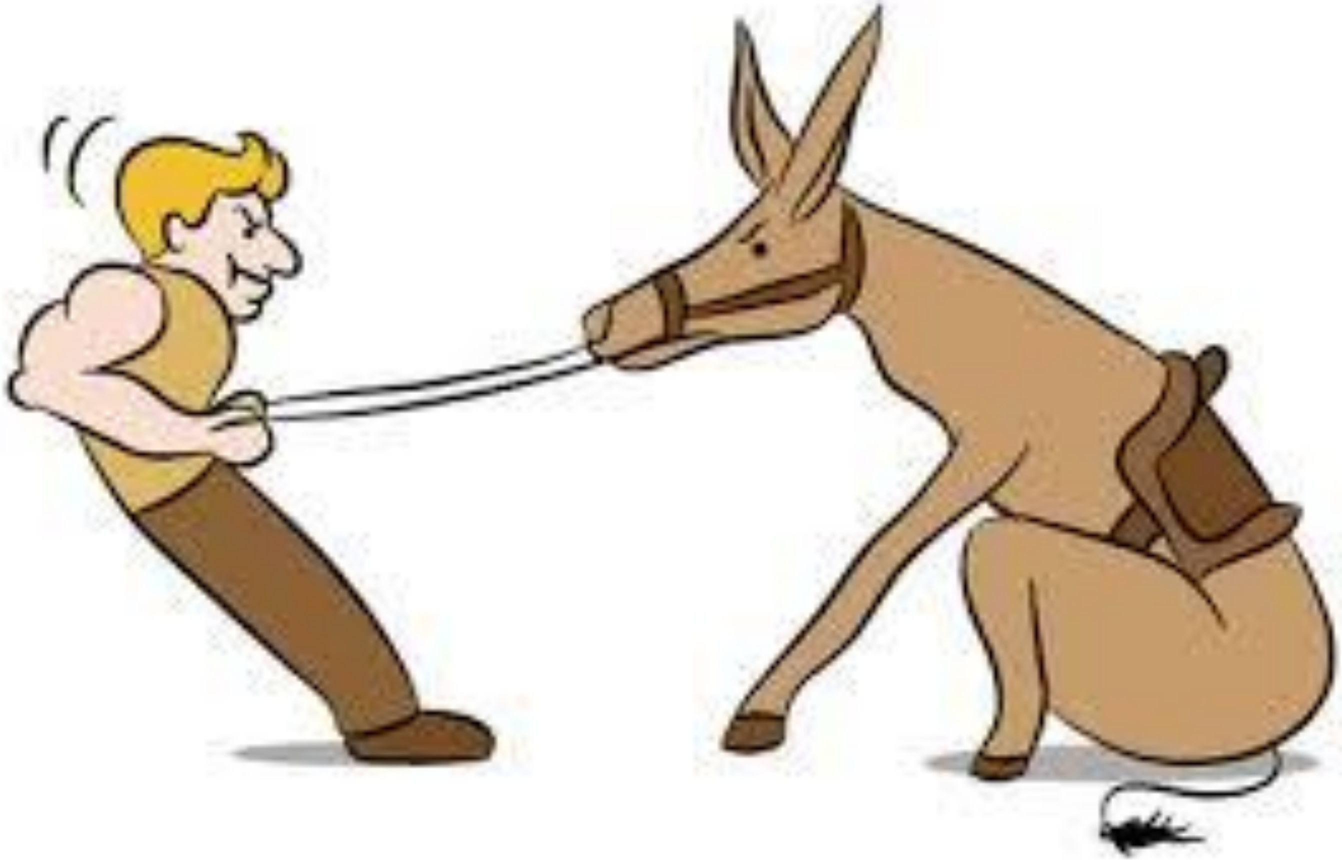
Fully friendship Network







Stubborns



Stubborn Relation: Rigid relationship between two agents.

Signed networks move towards balance, but in real networks there are some links with no tendency to change, though changing them would make the system tensionless.

Bigotry

The joy is wiser than wisdom

Enjoy your life!!??!?!?!?

Effect of stubborn nodes on network dynamics

The Stubborn nodes resistant to reduce energy

Question:

What is the minimum number of stubborn links to make a system stable in positive energy?

How many stubborn links could be existed and E<0. The society is tolerated them?

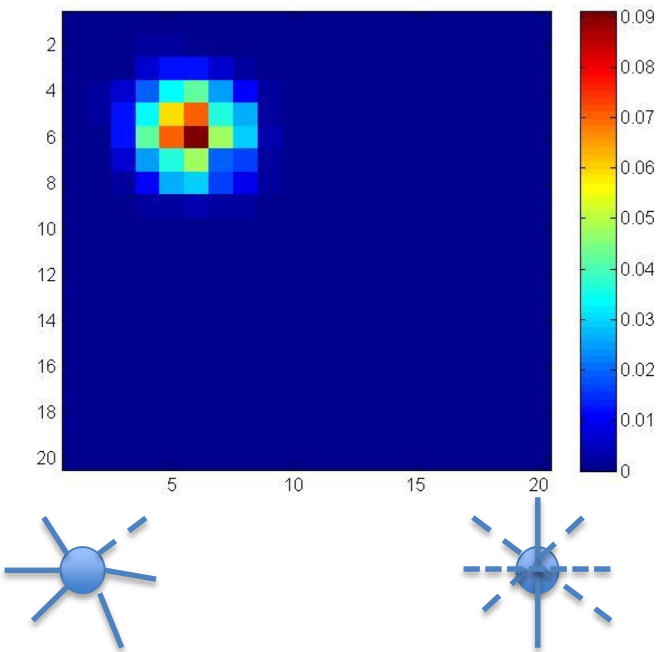
Our results show:

Minimum 17% links (about 4% nodes) are needed to make a unstable situation with E>0

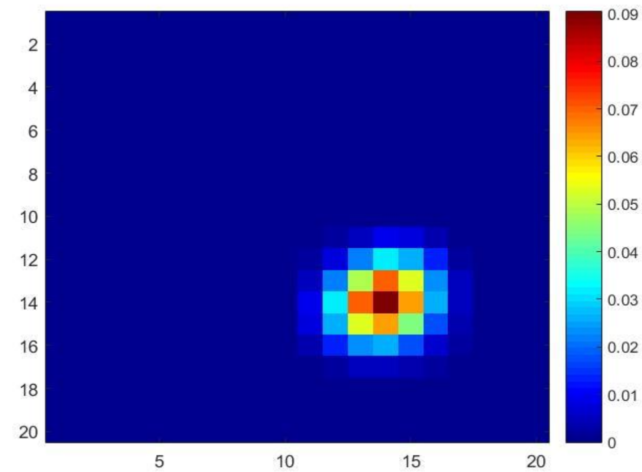
Maximum 83% links (about 9% nodes) could be existed in a stable situation with E<0.

Ranking stubborn nodes rank-rank correlation

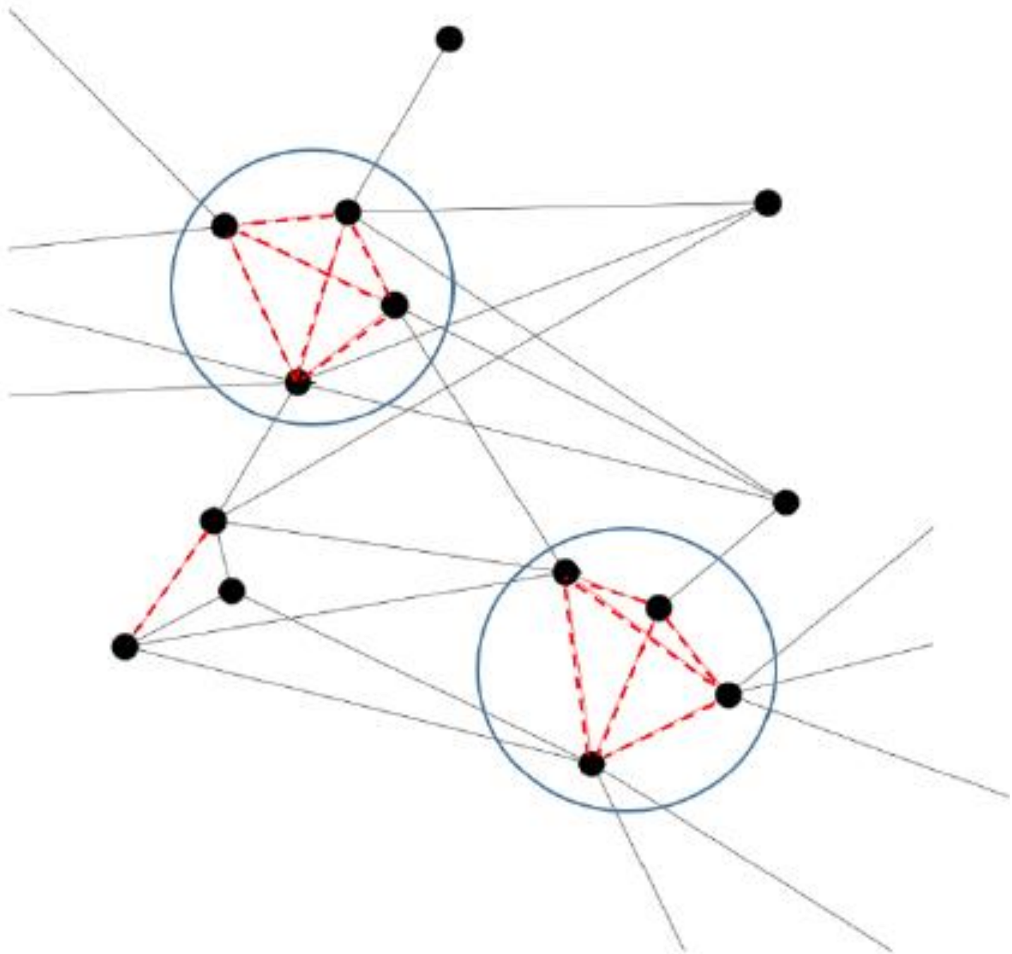
minimum



maximum



Anti-Community



Minimum Stubborn links in the networks with E<0

Anti-Community

$$All\ Triads = \binom{N}{3}$$

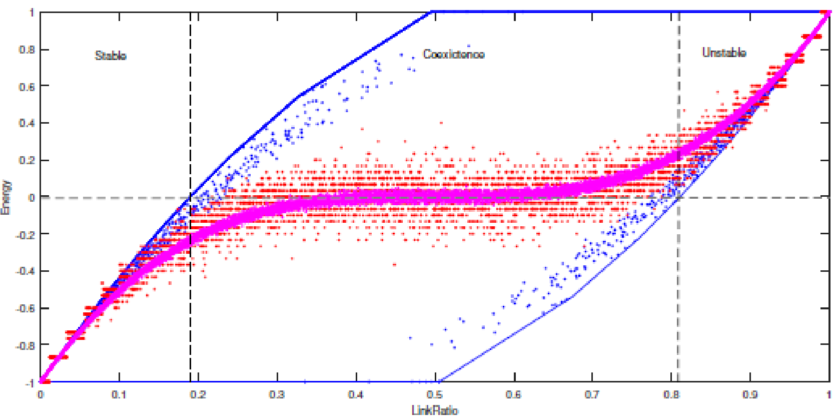
$$Energy > 0\ ;\ UnBallanced\ Triads\ - Ballanced\ Triads > 0$$

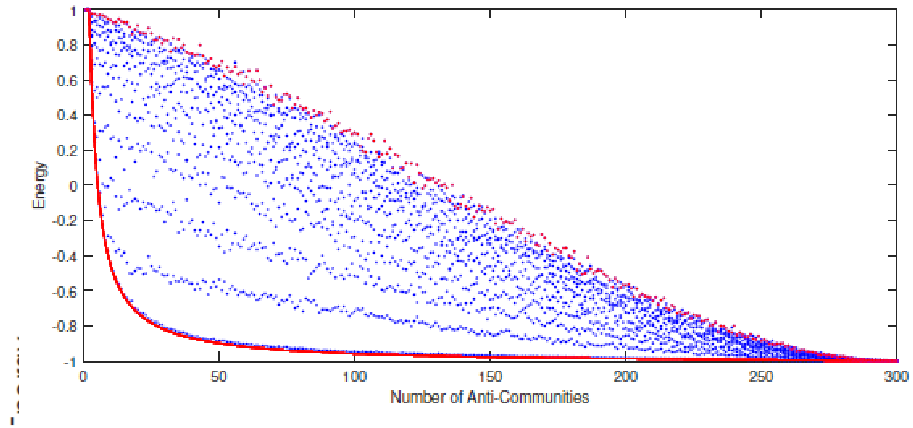
$$UnBallanced\ Traids = m[\binom{\frac{N}{m}}{1} + \binom{\frac{N}{m}}{3}]$$

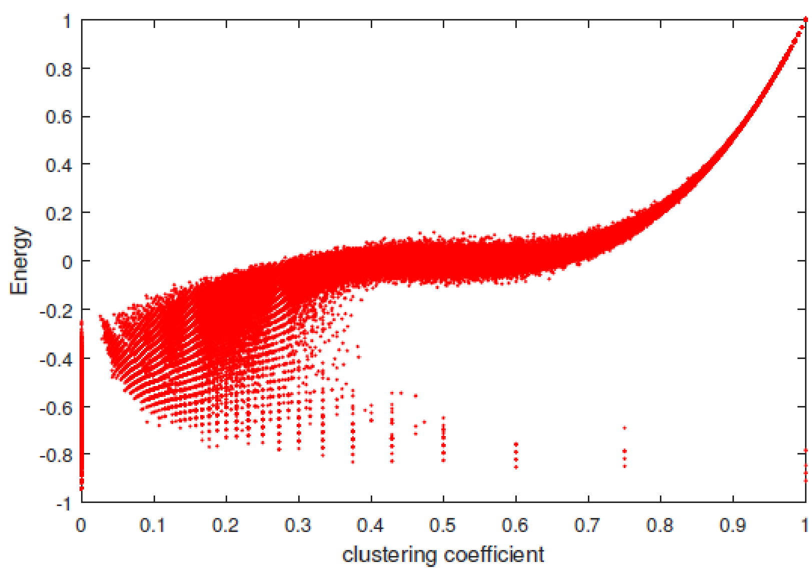
$$Condition\ for\ E > 0 \qquad m[\binom{\frac{N}{m}}{1} + \binom{\frac{N}{m}}{3}] \leq \frac{1}{2}\binom{N}{3}$$

Based on number of nodes live in the network $m \leq 3 \pm \sqrt{5}$

N : Node numbers;
m: Cluster numbers


Stable
Coexistence
Unstable
Energy
LinkRatio


Energy
Number of Anti-Communities


Energy
clustering coefficient